diner

$u_i = c_i \cdot c_i'$

grans	G1	0
	G2	2
joves	G1	1
	G2	2

50, 50, 50, 50

← en $t=0$ creen la quantitat M de diner (el diner només es crea un cop)

Sense mercat de préstecs

m_i = demanda de diner d'i

p = preu del diner en béns $\left(p \ \frac{\text{unitats monetàries}}{\text{unitats de béns}}\right)$

$\frac{m_i}{p}$ = demanda real de diner

G1

$\max_{c_1, c_1'} u_1 = c_1 \cdot c_1'$

s.a. $\begin{cases} c_1 + \frac{m_1}{p} = 1 \\ c_1' = \frac{m_1}{p'} \end{cases}$

$\downarrow$

$\max_{m_1} \left(1 - \frac{m_1}{p}\right)\left(\frac{m_1}{p'}\right)$

$0 = \frac{d}{dm_1} = \frac{1}{p'} - \frac{2m_1}{p \cdot p'}$

$\boxed{m_1 = p/2} \to c_1 = 1/2$

G2

$\max_{c_2, c_2'} u_2 = c_2 \cdot c_2'$

s.a. $\begin{cases} c_2 + \frac{m_2}{p} = 2 \\ c_2' = 2 + \frac{m_2}{p'} \end{cases}$

$\downarrow$

$\max_{m_2} \left(2 - \frac{m_2}{p}\right)\left(2 + \frac{m_2}{p'}\right)$

$0 = \frac{d}{dm_2} = \frac{2m_2}{p'} - \frac{2m_2}{p} + \frac{2m_2^2}{p \cdot p'}$

$\boxed{m_2 = \left(\frac{1}{p} - \frac{1}{p'}\right)pp' = p' - p}$

equilibri en el mercat de diner

$50 m_1 + 50 m_2 = M$

$50(m_1 + m_2) = M$

$50\left(\frac{p}{2} + p' - p\right) = M$

$50\left(p' - \frac{p}{2}\right) = M$

$50p' - 25p = M$ ← vàlid per a tot t (M constant)

$50p'' - 25p' = M$

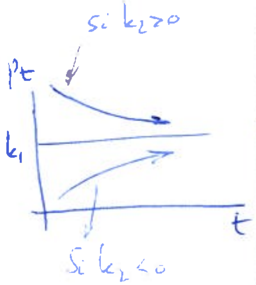
$50p' - 25p = 50p'' - 25p'$

$50p'' - 75p' + 25p = 0$

$\boxed{2p'' - 3p' + p = 0}$ $c_1 = c_1' = 1/2$

equació diferencial lineal

$c_2 = c_2' = 2$ ← $\boxed{p = \frac{M}{25}}$ ← $\frac{M}{50} = \frac{p}{2}$



$2d^2 - 3d + 1 = 0$

$d = \frac{3 \pm \sqrt{9-8}}{4} \begin{cases} 1 & \leftarrow d_1 \\ 1/2 & \leftarrow d_2 \end{cases}$

Solució general:

$p_t = k_1 \cdot d_1^t + k_2 \cdot d_2^t$, k_1, k_2 constants

$p_t = k_1 + k_2\left(\frac{1}{2}\right)^t$

Amb condicions inicials p_0, p_1

$p_0 = k_1 + k_2\left(\frac{1}{2}\right)^0 = k_1 + k_2$

$p_1 = k_1 + k_2\left(\frac{1}{2}\right)^1 = k_1 + \frac{k_2}{2}$

$\to p_0 - p_1 = k_2 - \frac{k_2}{2} = \frac{k_2}{2}$

$\boxed{k_2 = 2(p_0 - p_1)}$

$k_1 = p_0 - k_2 = p_0 - 2p_0 + 2p_1 = 2p_1 - p_0$

$p_t = (2p_1 - p_0) + 2(p_0 - p_1)\left(\frac{1}{2}\right)^t$

$t \to \infty \Rightarrow \boxed{p_t \to 2p_1 - p_0}$ → p_t eventualment constant

atès que $p_t \geq 0 \to 2p_1 \geq p_0 \to p_1 \geq p_0/2$

$p = p'$

$m_2 = 0$ $m_1 = \frac{M}{50}$

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