4. An overlapping generations model with Malthusian overtones

1. Economic regimes

Galor and Weil (1999, p. 150; 2000, p. 806) characterize the process of economic development in terms of three regimes. In historical order, they are called Malthusian, Post-Malthusian, and Modern Growth Regimes.

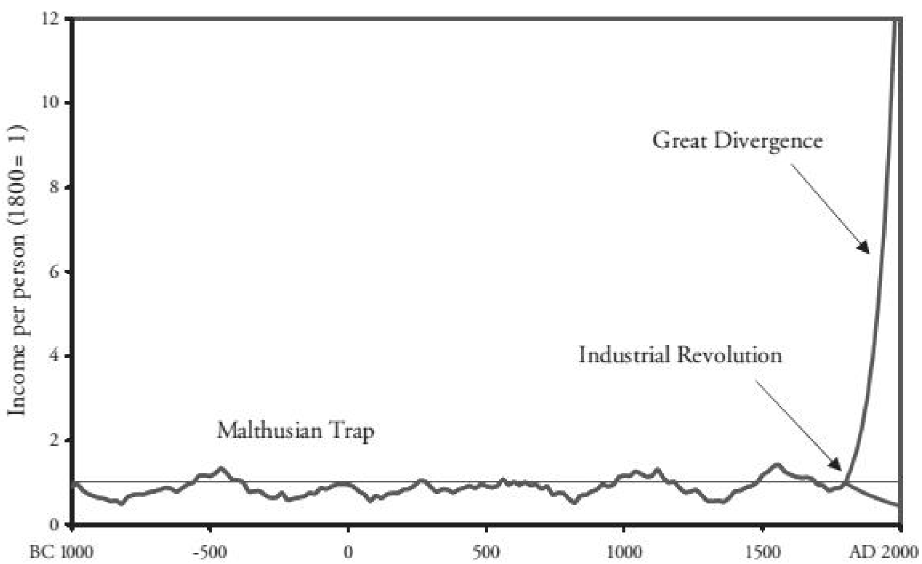
According to their characterization, in the Malthusian Regime technological progress and population growth was almost negligible ("glacial by modern standards"), whereas income per capita (living standards) was nearly constant. In addition, there exists a positive relationship between income per capita and population growth: an increase in per capita income leads to an increase in population growth.

In the Post-Malthusian Regime income per capita grows but the positive relationship between per capita income and population growth still holds: a rising income per capita continues to lead to a rising population growth rate.

Lastly, the Modern Growth Regime is the opposite of the Malthusian Regime: technological level and income per capita steadily grow, at a higher rate than in the Post-Malthusian Regime, and the relationship between the level of income per capita and the population growth rate turns out to be negative, in the sense that a rising income per capita leads to a declining population growth rate.

Clark (2007, p. 1) describes this view of the past as follows:

> The basic outline of world economic history is surprisingly simple. Indeed it can be summarized in one diagram [shown below, taken from Clark (2007, p. 2)]. Before 1800 income per person—the food, clothing, heat, light, and housing available per head—varied across societies and epochs. But there was no upward trend. A simple but powerful mechanism …, the Malthusian Trap, ensured that short-term gains in income through technological advances were inevitably lost through population growth. Thus the average person in the world of 1800 was no better off than the average person of 100,000 BC. Indeed in 1800 the bulk of the world's population was poorer than their remote ancestors.



Incidentally, Clark (2007, pp. 10-11) offers a bold explanation for the entry in the Modern Growth Regime (underlining added):

> Why an Industrial Revolution in England? Why not China, India, or Japan? The answer hazarded here is that England's advantages were not coal, not colonies, not the Protestant Reformation, not the Enlightenment, but the accidents of institutional stability and demography: in particular the extraordinary stability of England back to at least 1200, the slow growth of English population between 1300and 1760, and the <u>extraordinary fecundity of the rich and economically successful</u>. The embedding of bourgeois values into the culture, and perhaps even the genetics, was for these reasons the most advanced in England. Both China and Japan were headed in the same direction as England in 1600–1800… but they were headed there more slowly than England … simply because the members of their upper social strata were only modestly more fecund than the mass of the population. Thus there was not the same cascade of children from the educated classes down the social scale.

Allen (2008), Persson (2008), and Voth (2008) raise objections to Clark's claims. Voth (2008, pp. 150-151) replies to the fecundity explanation of England's rise as follows:

> The nobility in England and other parts of Europe was not particularly successful in reproducing… The Chinese rich often had concubines, in part with the aim of producing as many sons as possible. Simple arithmetic suggests that any polygamous society (in which taking another wife is tied to wealth) should have produced a much steeper fertility gradient by wealth than Europe did.

The contention that population growth ate up all productivity gains before 1800 is replied as well.

> One of the major flaws of the Malthusian hypothesis … is that it lacks a micro-foundation in human optimizing behaviour … The claim that an income increase would necessarily spill over into an increase in thenumberof children born entails an arbitrary restriction on the choice set of households. Why would the household spend the income on more children rather than fewer and better children or on other goods? Persson (2008, pp. 166-167)

> The most enduring empirical findings from preindustrial Europe were that population was growing – except for periods with exogenous shocks such as the Black Death – and that per capita income hence was above subsistence and differed substantially between economies. There were irrefutable signs that a few economies managed to increase average income over time slowly, for example the Netherlands, while others remained at a constant above subsistence income with positive population growth for centuries, for example Italy. Persson (2008, p. 171)

> Observing … that densely populated areas seemed to have higher income and productivity levels, it would have made sense to link the rate of technological progress to population as has been done recently in New Growth Theory. That would have turned the Malthusian story upside down and focused on what really constrains pre-industrial economies: slow technological progress, which might be stimulated by population growth! Persson (2008, p. 172)

> While it is widely believed that the preindustrial world was Malthusian, the view is controversial among economic historians. Allen (2008, p. 948)

> Clark's view of preindustrial England is oversimplified … he imagines a bipolar world of very rich and very poor … This vision ignores the importance of human capital in the preindustrial world and therefore misses the prosperity of the middle strata of preindustrial England. Ignoring the middle strata means overlooking much of the gain from the economic growth. The prosperity of the middle strata was due to an extensive division of labor within and between firms and to a high endowment of craft skills on which preindustrial technology depended. Human capital was essential to economic growth in the preindustrial era just as it became later. Allen (2008, p. 954)

> Contrary to Clark, our forebearers were not enjoying abundance in a Garden of Eden. Economic growth before the Industrial Revolution was not rapid, but it did generate a higher standard of living for most people than that enjoyed by ancient foragers or early farmers. Allen (2008, p. 955)

> Table 1 gives a summary of the kind of goods that the elderly English poor owned in the eighteenth century … By definition, the group of individuals to whom the data … refer is not rich. Yet 95% of them had chairs, half owned candlesticks and a fifth had a clock or watch. As Landes (2000, *Revolution in Time: Clocks and the Making of the Modern World*) points out, watches and clocks were so expensive that even by the middle of the twentieth century, they were widely considered luxury items. Their ancestors on the African savannah (with whom Clark compares the living standards of the English poor) would undoubtedly have marvelled at these possessions and would have thought them the pinnacle of luxury. Voth (2008, p. 152)

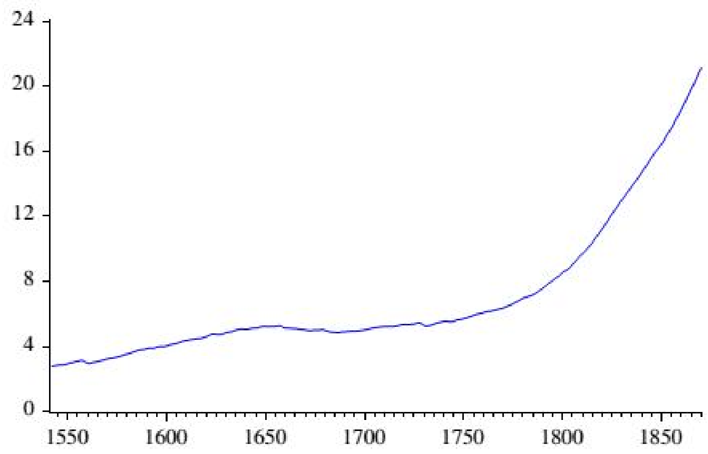
Table 1. *Consumer durables in Essex pauper inventories*

Item	% of households owning the item
Chairs	95
Chest of drawers	32
Mahogany furniture	5
Looking-glass	27
Clocks/watches	20
Pictures/prints	10
Candlesticks	49
Poker	12
Tea-related items	46

On the question of why the Malthusian myth still survives, Craig (2009, p. 95) remarks that

> Malthus remains part of the canon because subsequent generations of scholars and pundits invoked his name when forecasting economic and social catastrophes, which were the result – or, depending on your perspective, the failure – of certain policy prescriptions. Thus, whenever someone raises the specter of too many people and too little space or food, because of some policy, or the failure to adopt some other policy, Malthus is often implicated as the intellectual antecedent. But there is a difference between Hell and going to Hell, and the fact that most of the world is not Hell most of the time has meant that Malthus (or, more accurately, subsequent generations of Malthusians) has become a prime example of how wrong economists can be. In recent decades, advances in econometrics combined with new time series of historical data have created the potential for a brave new world of Malthusian scholarship.

Craig (2009) reviews Crafts and Mills (2009), who "confirm that [pre-industrial England] was a Malthusian economy in the sense that real wages were stationary until the end of the eighteenth century". The graph on the right, taken from Crafts and Mills (2009, p. 72), represents the population of England (in millions) from 1541 to 1870. How plausible is the claim that, before 1800, this corresponds to the population of an economy that is subject to the Malthusian trap? The graph suggests that some new dynamics (that manifests itself in an exponential population growth) already started around 1650.



2. A Malthusian model

Ashraf and Galor (2011, pp. 2005-09) suggest a model that replicates the basic features of the Malthusian Regime. The model is described next.

Only one good Y is produced in the economy, which operates in discrete time. There are two inputs: land D and labour L. The amount of land is fixed and constant in every period. The production function of the economy in period t is

$$Y(t) = (A \cdot D)^{\alpha} \cdot L(t)^{1-\alpha}$$

where:

- $Y(t)$ is the amount of the good produced in period t;
- A represents the state of the technology (the technology "level");
- D is the fixed amount of land;
- α is a number between 0 and 1; and
- $L(t)$ is the amount of labour employed in production in period t.

The term $A \cdot D$ captures the resources that are effectively used in production. Loosely speaking, the state of the technology represents all the factors that make land more or less productive: soil quality, climate, cultivation methods, knowledge of techniques to apply land to production, etc. The combination $A \cdot D$ define the effective land resources used in production: if $A = 2$, then having one unit of land is, in effect, like having two.

The agents providing labour are called farmers. The output per farmer $y(t)$ in period t is defined as

$$y(t) = \frac{Y(t)}{L(t)} = \left(\frac{A \cdot D}{L(t)}\right)^{\alpha}.$$

Individuals live for two consecutive periods. All individuals are identical. In the second period of life (parenthood) every individual:

- decides how many children to have (families are single-parent);
- becomes a farmer and supplies labour inelastically (the farmer's supply of labour does not depend on income);
- the farmer's income in period t coincides with the output per farmer $y(t)$ in period t;
- a farmer's income is spent in consumption and in raising children.

In the first period of life (childhood), every individual is a child and, therefore, must be supported by his/her parent. As a child, an individual makes no decision.

The utility function $u(t)$ of an individual in his/her second period of life t is given by

$$u(t) = c(t)^{\beta} \cdot n(t)^{1-\beta}$$

where $c(t)$ is the amount of the good consumed by the individual in period t, $n(t)$ is the number of children that the individual has chosen to have, and β is a number between 0 and 1.
Raising a child has a fixed cost of $\gamma > 0$ units of the good per child. Each farmer from period t chooses $c(t)$ and $n(t)$ in order to maximize $u(t)$ facing the following budget constraint:

$$c(t) + \gamma \cdot n(t) = y(t).$$

To solve the problem

$$max_{c(t),n(t)}\ u(t) = c(t)^{\beta} \cdot n(t)^{1-\beta}$$
$$\text{subject to } c(t) + \gamma \cdot n(t) = y(t)$$

using the Lagrange multiplier techique, define

$$L(t) = c(t)^{\beta} \cdot n(t)^{1-\beta} + \lambda \cdot \big(y(t) - c(t) - \gamma \cdot n(t)\big).$$

Then:

$$\frac{\partial L(t)}{\partial c(t)} = \beta \cdot c(t)^{\beta-1} \cdot n(t)^{1-\beta} - \lambda = 0$$

$$\frac{\partial L(t)}{\partial n(t)} = (1-\beta) \cdot n(t)^{-\beta} \cdot c(t)^{\beta} - \lambda \cdot \gamma = 0\,.$$

By the first equation,

$$\lambda = \beta \cdot c(t)^{\beta-1} \cdot n(t)^{1-\beta}.$$

By the second,

$$\lambda = \frac{1-\beta}{\gamma} \cdot c(t)^{\beta} \cdot n(t)^{-\beta}.$$

Therefore,

$$\beta \cdot c(t)^{\beta} \cdot \frac{1}{c(t)} \cdot n(t)^{-\beta} \cdot n(t) = \frac{1-\beta}{\gamma} \cdot c(t)^{\beta} \cdot n(t)^{-\beta}$$

and, consequently,

$$n(t) = \frac{1-\beta}{\beta \cdot \gamma} \cdot c(t)\,.$$

Inserting this into the budget constraint $c(t) + \gamma \cdot n(t) = y(t)$ yields

$$\boxed{c(t) = \beta \cdot y(t)\,.}$$

As a result,

$$\boxed{n(t) = \frac{1-\beta}{\gamma} \cdot y(t)\,.}$$

Since $1-\beta > 0$, an increase in output per farmer leads to an increase in the number of children: $\frac{dn(t)}{dy(t)} = \frac{1-\beta}{\gamma} > 0$. The model then reproduces the positive relationship between output per capita and population growth that corresponds to the Malthusian Regime.

3. Population dynamics and stationary state

For period t, let $L(t)$ designate the number of farmers in t (the working population). Consequently,

$$L(t+1) = n(t) \cdot L(t)\,.$$

This says that the number of famers in $t+1$ corresponds to the number of children that the farmers in t decided to have: the children of period t farmers become period $t+1$ farmers.

By combining the equations $L(t+1) = n(t) \cdot L(t)$, $n(t) = \frac{1-\beta}{\gamma} \cdot y(t)$, and $y(t) = \left(\frac{A \cdot D}{L(t)}\right)^{\alpha}$, it follows that

$$L(t+1) = \frac{1-\beta}{\gamma} \cdot \left(\frac{A \cdot D}{L(t)}\right)^{\alpha} \cdot L(t)$$

that is,

$$L(t+1) = \frac{1-\beta}{\gamma} \cdot (A \cdot D)^{\alpha} \cdot L(t)^{1-\alpha}$$

or, letting $a = \frac{1-\beta}{\gamma} \cdot (A \cdot D)^{\alpha}$,

$$\boxed{L(t+1) = a \cdot L(t)^{1-\alpha}\,.}$$

Since $a > 0$ and $0 < 1 - \alpha < 1$,

$$\frac{dL(t+1)}{dL(t)} = a \cdot (1-\alpha) \cdot L(t)^{-\alpha} = \frac{a \cdot (1-\alpha)}{L(t)^{\alpha}} > 0$$

and

$$\frac{d^2L(t+1)}{dL(t)^2} = -a \cdot (1-\alpha) \cdot \alpha \cdot L(t)^{-\alpha-1} < 0\,.$$

The above derivatives mean that the function represented by $L(t+1) = a \cdot L(t)^{1-\alpha}$ is:

(i) increasing;
(ii) strictly concave;
(iii) intersects the origin ($L(t) = 0$ implies $L(t+1) = 0$); and
(iv) the first derivative $\frac{dL(t+1)}{dL(t)}$
  (a) is decreasing;
  (b) approaches zero as $L(t)$ goes to infinity; and
  (c) goes to infinity as $L(t)$ converges to zero.

Fig. 1 below draws $L(t+1) = a \cdot L(t)^{1-\alpha}$. Observe that, apart from the origin, the graph intersects the main diagonal (that is, the set of pairs $\big(L(t+1), L(t)\big)$ such that $L(t+1) = L(t)$) only once: at point e. At that point, corresponding to the value L^*, a <u>stationary</u> (or steady) <u>state</u> is reached, in the sense that $L(t) = L^*$ implies $L(t+1) = L^*$.

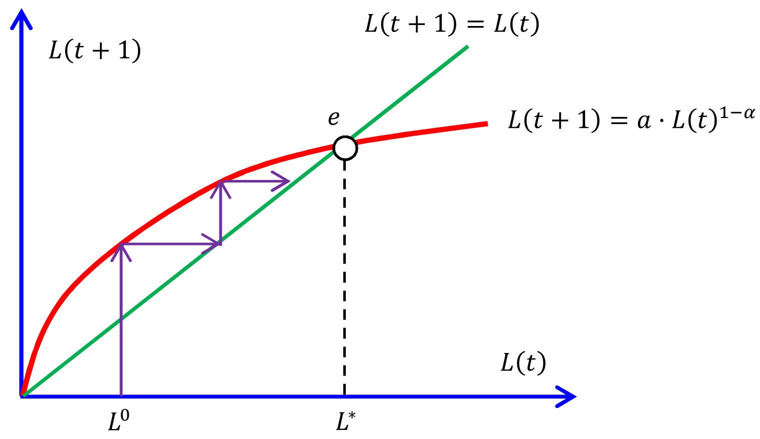


Fig. 1. Population dynamics and population steady state

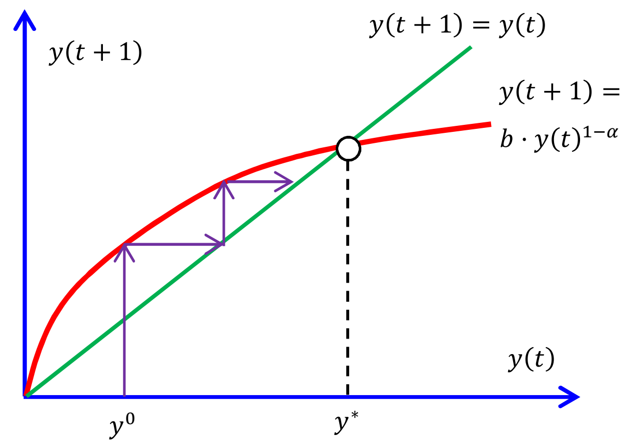


Fig. 2. Output per capita

The only stationary state (different from zero) associated with $L(t+1) = a \cdot L(t)^{1-\alpha}$ is obtained by setting $L(t+1) = L(t)$ and satisfies

$$L^* = a \cdot {L^*}^{1-\alpha}.$$

Solving for L^* yields the stationary value of population: $L^* = a^{1/\alpha}$. That is,

$$\boxed{L^* = A \cdot D \cdot \left(\frac{1-\beta}{\gamma}\right)^{\frac{1}{\alpha}}.} \tag{1}$$

The stationary state is <u>stable</u> in the same that, if the initial value of population is different from the stationary value L^*, the dynamics of the process $L(t+1) = a \cdot L(t)^{1-\alpha}$ makes population converge to L^*. This is illustrated in Fig. 1 by assuming that the initial value is $L^0 < L^*$: the sequence of values generated by the process approaches L^*. The same conclusion holds if $L^0 > L^*$.

Define the population density (or, properly, the farmers' density) in period t as $F(t) = L(t)/D$. The stationary density is then given by $F^* = L^*/D$ or

$$\boxed{F^* = A \cdot \left(\frac{1-\beta}{\gamma}\right)^{\frac{1}{\alpha}}.} \tag{2}$$

4. Output per capita dynamics and stationary state

Recalling that $y(t) = \left(\frac{A \cdot D}{L(t)}\right)^{\alpha}$ and $L(t+1) = n(t) \cdot L(t)$,

$$y(t+1) = \left(\frac{A \cdot D}{L(t+1)}\right)^{\alpha} = \left(\frac{A \cdot D}{n(t) \cdot L(t)}\right)^{\alpha} = \frac{1}{n(t)^{\alpha}} \cdot \left(\frac{A \cdot D}{L(t)}\right)^{\alpha} = \frac{y(t)}{n(t)^{\alpha}}.$$

Since $n(t) = \frac{1-\beta}{\gamma} \cdot y(t)$, it turns out that the following first-order difference equation describes the dynamics of output per capita:

$$y(t+1) = \left(\frac{\gamma}{1-\beta}\right)^{\alpha} \cdot y(t)^{1-\alpha}. \tag{3}$$

Setting $b = \left(\frac{\gamma}{1-\beta}\right)^{\alpha} > 0$, this can be expressed in a more compact form:

$$\boxed{y(t+1) = b \cdot y(t)^{1-\alpha}.}$$

This expression is analogous to the one describing the dynamics of the working population: $L(t+1) = a \cdot L(t)^{1-\alpha}$. See Fig. 2 for a graphical representation.

5. Exercicis

Exercici 1. Estabilitat d'un estat estacionari. Explica si és estable l'estat estacionari representat per l'origen de la Fig. 1 (no hi ha població). Això és, assegura la dinàmica de la població el retorn a l'origen si es produeix una pertorbació que allunya mínimament l'economia de l'origen?

Exercici 2. Estàtica comparativa. Partint de la Fig. 1: (a) analitza gràficament l'impacte sobre l'estat estacionari (i, en particular, sobre L^* i F^*) de cadascun dels següents esdeveniments; (b) confirma el resultat de l'anàlisi gràfica determinant el signe de la derivades parcials corresponent d'(1) i (2); (c) indica què succeeix amb la renta per càpita d'estat estacionari.

(i) Es produeix una millora tecnològica (augmenta el valor del paràmetre A).

(ii) Es produeix una regressió tecnològica (disminueix el valor del paràmetre A).

(iii) S'incrementa el cost de tenir fills.

(iv) Augmenta l'estoc de terra D (descobriment d'Amèrica).

(v) Creix la preferència pel consum (s'apuja el valor del paràmetre β).

(vi) Es redueix el valor del paràmetre α.

Exercici 3. Teorema d'Euler. Pren la funció de producció estàtica $Y = (A \cdot D)^{\alpha} \cdot L^{1-\alpha}$.

(i) Calcula la funció $\frac{\partial Y}{\partial D}$ de productivitat marginal de la terra.

(ii) Calcula la funció $\frac{\partial Y}{\partial L}$ de productivitat marginal dels pagesos.

(iii) Verifica que $Y = \frac{\partial Y}{\partial D} \cdot D + \frac{\partial Y}{\partial L} \cdot L$.

(iv) Ofereix una interpretació econòmica de l'equació $Y = \frac{\partial Y}{\partial D} \cdot D + \frac{\partial Y}{\partial L} \cdot L$.

(v) Determina $\frac{\frac{\partial Y}{\partial D} \cdot D}{Y}$ i interpreta el resultat.

(vi) Assumint $A = 1$, troba la derivada de la funció $\frac{\partial Y}{\partial D}$ respecte d'α i interpreta econòmicament el signe.

Exercici 4. Dinàmica de la renda per càpita. Troba l'expressió que defineix l'estat estacionari (no trivial) de la dinàmica de la renda per càpita representada per (3) i determina el signe de la derivada parcial respecte de cada paràmetre.

Exercici 5. Prediccions malthusianes. Estableix si les següents prediccions malthusianes es produeixen en el model explicat al text. [Ashraf i Galor (2011, p. 2009) assenyalen que "These predictions emerge from a Malthusian model as long as the model is based upon two fundamental features: (i) a positive effect of the standard of living on population growth, and (ii) decreasing returns to labor due to the presence of a fixed factor of production—land."]

(i) Una millora tecnològica que es tradueix en una augment de la productivitat de la terra (augment d'A) fa que, en el llarg termini, s'incrementi la població sense que es modifiqui el nivell de la renda per càpita.

(ii) Si l'única diferència estructural entre dues economies E i E' és que la productivitat de la terra (o el nivell tecnològic) d'E és superior a la d' E', aleshores, a llarg termini, l'economia E tindrà una densitat de població superior a la d'E' però no tindrà una renda per càpita més gran que E'.

Exercici 6 (voluntari). Extenent el model. El model obté una relació positiva entre el nombre de fills n i la renda per càpita y. Redefineix el model (de manera convincent i raonada) amb l'objectiu d'aconseguir el següent resultat: per a valors d'y inferiors a un cert valor y', la relació entre n i y és positiva, però per a valors superiors a y' la relació és negativa (un augment d'y causa una reducció d'n).

Exercici 7. Malthus dixit. Opina sobre el següent text:

> "All the children born, beyond what would be required to keep up the population to this level, must necessarily perish, unless room be made for them by the deaths of grown persons … the marriages and births depend principally upon the deaths, and … there is no encouragement to early unions so powerful as a great mortality. To act consistently therefore, we should facilitate, instead of foolishly and vainly endeavouring to impede, the operations of nature in producing this mortality… Instead of recommending cleanliness to the poor, we should encourage contrary habits. In our towns we should make the streets narrower, crowd more people into the houses, and court the return of the plague. In the country, we should build our villages near stagnant pools, and particularly encourage settlements in all marshy and unwholesome situations. But above all, we should reprobate specific remedies for ravaging diseases… If … the annual mortality were increased from 1 in 36 or 40, to 1 in 18 or 20, we might probably every one of us marry at the age of puberty, and yet few be absolutely starved. If, however, we all marry at this age, and yet still continue our exertions to impede the operations of nature, we may rest assured that all our efforts will be vain. Nature will not, nor cannot, be defeated in her purposes. The necessary mortality must come, in some form or other; and the extirpation of one disease will only be the signal for the birth of another perhaps more fatal." Rev. T. R. Malthus (1826): *An Essay on the Principle of Population*, Vol. II, 6th ed., John Murray, London, pp. 300-302 (Book IV, ch. V)

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