9. Monetary policy in aggregate supply and demand models

1. A model where monetary policy is irrelevant

Variables are measured in natural logarithms. The short-run aggregate supply (AS) function of the economy is

$$y_t = y^* + \alpha \cdot (p_t - E_{t-1}p_t) + u_t \tag{1}$$

where $\alpha > 0$, y^* is potential GDP, p_t is the price level, $E_{t-1}p_t$ is the price level in period t that is expected in $t-1$ (using 'efficiently' the information in $t-1$), and $u_t \sim N(0, \sigma_u^2)$ is an independent random variable. The logic of equation (1) is that if the price level is understimated (so $p_t > E_{t-1}p_t$), then too much labour is supplied and output expands above potential. The short-run aggregate demand (AD) function is

$$y_t = a + \beta \cdot (m_t - p_t) + \beta' \cdot E_{t-1}(p_{t+1} - p_t) + v_t \tag{2}$$

where $\beta, \beta' > 0$, the real balance term $m_t - p_t$ captures the LM relation (the Keynes effect), the expected inflation rate $E_{t-1}(p_{t+1} - p_t)$ represents a Tobin effect, and v_t is an independent random variable $v_t \sim N(0, \sigma_v^2)$ uncorrelated with u_t: $E(u_t, v_t) = 0$. An interpretation of (2) is that a higher rate of expected inflation implies a lower real interest rate, a higher investment rate, and a higher aggregate demand.

The monetary authority is supposed to follow the monetary rule

$$m_t = \gamma_0 + \gamma_1 \cdot m_{t-1} + \gamma_2 \cdot y_{t-1} + z_t \tag{3}$$

where z_t is an independent random variable $u_t \sim N(0, \sigma_u^2)$, uncorrelated with u_t and v_t, that captures the imperfect control of the monetary authority over monetary aggregates. Monetarists would set $\gamma_1 = \gamma_2 = 0$ (constant money supply) or, at most, $\gamma_1 > 0$. A Keynesian would prefer $\gamma_1 \geq 0$ and $\gamma_2 < 0$ (money supply raised to stimulate output).

The model can be solved in five steps.

• **Step 1**: equate AS and AD and solve for p_t.

$$p_t = \frac{a - y^* + \beta \cdot m_t + \alpha \cdot E_{t-1}p_t + \beta' \cdot E_{t-1}(p_{t+1} - p_t) + u_t + v_t}{\alpha + \beta}$$

• **Step 2**: take the expectation of p_t in $t-1$.

$$E_{t-1}p_t = \frac{a - y^* + \beta \cdot E_{t-1}m_t + \alpha \cdot E_{t-1}E_{t-1}p_t}{\alpha + \beta} + \frac{\beta' \cdot E_{t-1}E_{t-1}(p_{t+1} - p_t) + E_{t-1}u_t + E_{t-1}v_t}{\alpha + \beta}$$

Shocks are independent of themselves (not autocorrelated): $E_{t-1}u_t = E_{t-1}v_t = 0$. Moreover, $E_{t-1}E_{t-1}p_t = E_{t-1}p_t$ and $E_{t-1}(c \cdot x_t) = c \cdot E_{t-1}x_t$.

In sum,

$$E_{t-1}p_t = \frac{\alpha - y^* + \beta \cdot E_{t-1}m_t + \alpha \cdot E_{t-1}p_t + \beta' \cdot E_{t-1}(p_{t+1} - p_t)}{\alpha + \beta}.$$

• **Step 3**: compute $p_t - E_{t-1}p_t$.

$$p_t - E_{t-1}p_t = \frac{\beta}{\alpha + \beta}(m_t - E_{t-1}m_t) + \frac{1}{\alpha + \beta}(v_t - u_t)$$

Price surprises ($p_t \neq E_{t-1}p_t$) come only from unanticipated changes in the money supply or unexpected shocks to AD or AS.

• **Step 4**: insert the policy rule. Since $E_{t-1}m_t = \gamma_0 + \gamma_1 \cdot E_{t-1}m_{t-1} + \gamma_2 \cdot E_{t-1}y_{t-1} + E_{t-1}z_t = \gamma_0 + \gamma_1 \cdot m_{t-1} + \gamma_2 \cdot y_{t-1}$,

$$p_t - E_{t-1}p_t = \frac{\beta}{\alpha + \beta}z_t + \frac{1}{\alpha + \beta}(v_t - u_t)$$

• **Step 5**: substitute into AS.

$$y_t = y^* + \frac{\beta}{\alpha + \beta}u_t + \frac{\alpha}{\alpha + \beta}v_t + \frac{\alpha \cdot \beta}{\alpha + \beta}z_t \qquad (4)$$

Equation (4) is the stochastic steady-state solution for output, where u_t captures the random supply shocks, v_t the random demand shocks, and z_t factors affecting the money supply that the central bank cannot control. As there is no policy rule parameter in (4), policy is ineffective at influencing output.

2. A model where monetary policy is relevant

Workers sign two-period nominal wage contracts. In period t, half of the workforce is on the wage contract signed in $t-2$ running from $t-1$ to t and the other half on those signed in $t-1$ valid from t to $t+1$.

Let w_t^s represent the (logarithm of the) nominal wage in t in the contract signed at $s \in \{t-2, t-1\}$

• **Wage setting rule** $\quad w_t^s = E_s p_t$

• **AD function** $\quad y_t = m_t - p_t$

Firms are identical. In 50% of them, workers are on their first year contract. In the other 50%, workers are on their second (last) year.

• **AS function** $\quad y_t = \frac{1}{2}(p_t - w_t^{t-1} + u_t) + \frac{1}{2}(p_t - w_t^{t-2} + u_t) = \frac{1}{2}(p_t - E_{t-1}p_t) + \frac{1}{2}(p_t - E_{t-2}p_t) + u_t$

After equating AS and AD and solving for p_t

$$p_t = \frac{1}{2}\left(m_t - u_t + \frac{1}{2}(E_{t-1}p_t + E_{t-2}p_t)\right). \qquad (5)$$

Taking expectations conditional on $t-2$,

$$E_{t-2}p_t = \frac{1}{2}\left(E_{t-2}m_t + \frac{1}{2}(E_{t-2}p_t + E_{t-2}p_t)\right)$$

because $E_{t-2}E_{t-1}p_t = E_{t-2}p_t$. Therefore, $E_{t-2}p_t = E_{t-2}m_t$. Taking expectations conditional on $t-1$,

$$E_{t-1}p_t = \frac{1}{2}\left(E_{t-1}m_t + \frac{1}{2}(E_{t-1}p_t + E_{t-2}p_t)\right) = \frac{1}{2}\left(E_{t-1}m_t + \frac{1}{2}(E_{t-1}p_t + E_{t-2}m_t)\right).$$

Solving for $E_{t-1}p_t$ yields

$$E_{t-1}p_t = \frac{2}{3}E_{t-1}m_t + \frac{1}{3}E_{t-2}m_t\,.$$

- **Monetary rule** $m_t = \mu u_{t-1}$
- **Autocorrelated shock** $u_t = \rho u_{t-1} + \varepsilon_t$, with $|\rho| < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

$E_{t-1}m_t = \mu \cdot E_{t-1}u_{t-1} = \mu \cdot E_{t-1}[\rho \cdot u_{t-2} + \varepsilon_{t-1}] = \mu \cdot \rho \cdot E_{t-1}u_{t-2} = \mu \cdot \rho \cdot u_{t-2} = \mu \cdot (u_{t-1} - \varepsilon_{t-1}) = m_t - \mu \cdot \varepsilon_{t-1}$.

$E_{t-2}m_t = \mu \cdot E_{t-2}u_{t-1} = \mu \cdot E_{t-2}[\rho \cdot u_{t-2} + \varepsilon_{t-1}] = \mu \cdot \rho \cdot E_{t-2}u_{t-2} = \mu \cdot \rho \cdot E_{t-2}[\rho \cdot u_{t-3} + \varepsilon_{t-2}] = \mu \cdot \rho \cdot (\rho u_{t-3}) = \mu \cdot \rho \cdot (u_{t-2} - \varepsilon_{t-2}) = \mu \cdot (u_{t-1} - \varepsilon_{t-1}) - \mu \cdot \rho \cdot \varepsilon_{t-2} = m_t - \mu \cdot \varepsilon_{t-1} - \mu \cdot \rho \cdot \varepsilon_{t-2}$.

$E_{t-1}p_t + E_{t-2}p_t = \frac{2}{3}E_{t-1}m_t + \frac{4}{3}E_{t-2}m_t = 2 \cdot m_t - 2 \cdot \mu \cdot \varepsilon_{t-1} - \frac{4}{3} \cdot \mu \cdot \rho \cdot \varepsilon_{t-2}$. Inserting this into (5),

$$p_t = \frac{1}{2}\left(m_t - u_t + \left(m_t - \mu \cdot \varepsilon_{t-1} - \frac{2}{3} \cdot \mu \cdot \rho \cdot \varepsilon_{t-2}\right)\right)$$

or

$$p_t = m_t - \frac{u_t}{2} - \mu\left(\frac{\varepsilon_{t-1}}{2} + \rho\frac{\varepsilon_{t-2}}{3}\right).$$

After substituting the above expression into the AD function,

$$y_t = m_t - p_t = \frac{u_t}{2} + \mu\left(\frac{\varepsilon_{t-1}}{2} + \rho\frac{\varepsilon_{t-2}}{3}\right).$$

This proves that output depends on the policy rule parameter μ. The intuition is that, while the two-period contracts are in effect, there is room for the government to react to new events that, when contracts were signed, were not foreseeable or anticipated. Hence, half of the workers have signed contracts with outdated information.

3. A model for the design of a central bank

Imagine that $U_t = -\frac{1}{2}[\pi_t{}^2 + \alpha \cdot (y_t - \bar{y})^2]$ is a utility function that can be ascribed to a society, where $\bar{y}$ is the desired GDP, π is the inflation rate, and (in logs) y is real GDP. The AS function is the expression $y_t = y^* + \beta \cdot (\pi_t - \pi_t^e) + u_t$, where y^* is potential GDP, π^e the expected inflation rate, and u_t a random variable with mean value 0 and variance σ^2 that captures supply and demand shocks on the economy.

The utility function of the central bank (CB) is given by $U_t^{CB} = -\frac{1}{2}[{\pi_t}^2 + \gamma \cdot (y_t - \bar{y})^2]$ (it is presumed in U_t and U_t^{CB} that the inflation goal $\bar{\pi}$ is zero). The CB chooses π_t to maximize U_t^{CB}. Let the government have the power to pick γ (the extent to which the CB should care about the gap between output and desired output).

• **Option 1: $\boldsymbol{\gamma = 0}$**. This means that the CB only cares about inflation. Thus, $U_t^{CB} = -\frac{1}{2}{\pi_t}^2$ and $EU_t^{CB} = -\frac{1}{2}E{\pi_t}^2 = -\frac{1}{2}{\pi_t}^2$. Therefore, CB sets $\pi_t = 0$. This implies $\pi_t^e = E\pi_t = 0$ and, as a result,

$$EU_t^1 = -\frac{1}{2}\left[E{\pi_t}^2 + \alpha E(y_t - \bar{y})^2\right] = -\frac{\alpha}{2}E[y^* + \beta(\pi_t - \pi_t^e) + u_t - \bar{y}]^2 = -\frac{\alpha}{2}E[y^* - \bar{y} + u_t]^2$$
$$= -\frac{\alpha}{2}[(y^* - \bar{y})^2 + \sigma^2].$$

• **Option 2: $\boldsymbol{\gamma = \alpha}$**. That is, the preferences imposed on the CB are the society's. Then (assuming π_t^e independent of π_t):

$$0 = \frac{\partial U_t^{CB}}{\partial \pi_t} = -\pi_t - \alpha\beta^2(\pi_t - \pi_t^e) - \alpha\beta(y^* - \bar{y} + u_t)$$

so

$$\pi_t = \frac{\alpha\beta^2\pi_t^e - \alpha\beta(y^* - \bar{y} + u_t)}{1 + \alpha\beta^2}. \tag{6}$$

Taking expectations,

$$\pi_t^e = E\pi_t = \frac{\alpha\beta^2 E\pi_t^e - \alpha\beta E(y^* - \bar{y}) - \alpha\beta E u_t}{1 + \alpha\beta^2} = \frac{\alpha\beta^2\pi_t^e - \alpha\beta(y^* - \bar{y})}{1 + \alpha\beta^2}.$$

Solving for π_t^e, $\pi_t^e = \alpha\beta(\bar{y} - y^*)$. By (6),

$$\pi_t = \frac{\alpha\beta^2\alpha\beta(\bar{y} - y^*) + \alpha\beta(\bar{y} - y^*) - \alpha\beta u_t}{1 + \alpha\beta^2} = \alpha\beta(\bar{y} - y^*) - \frac{\alpha\beta}{1 + \alpha\beta^2} \cdot u_t.$$

Thus, $\pi_t - \pi_t^e = \frac{\alpha\beta}{1+\alpha\beta^2} \cdot u_t$. By the AS function,

$$y_t = y^* - \beta \cdot \frac{\alpha\beta u_t}{1 + \alpha\beta^2} + u_t = y^* + \frac{u_t}{1 + \alpha\beta^2}.$$

All in all, since $E{u_t}^2 = \sigma^2$,

$$EU_t^2 = -\frac{1}{2}\left[E{\pi_t}^2 + \alpha E(y_t - \bar{y})^2\right] = -\frac{1}{2}\left[E\left(\alpha\beta(\bar{y} - y^*) - \frac{\alpha\beta u_t}{1 + \alpha\beta^2}\right)^2 + \alpha \cdot E\left(y^* + \frac{u_t}{1 + \alpha\beta^2} - \bar{y}\right)^2\right] =$$
$$= -\frac{1}{2}\left[(\alpha^2\beta^2 + \alpha)(\bar{y} - y^*)^2 + \frac{\alpha^2\beta^2 + \alpha}{(1 + \alpha\beta^2)^2} \cdot \sigma^2\right] = -\frac{\alpha}{2}\left[(1 + \alpha\beta^2)(\bar{y} - y^*)^2 + \frac{(1 + \alpha\beta^2)}{(1 + \alpha\beta^2)^2} \cdot \sigma^2\right] =$$
$$= -\frac{\alpha}{2}\left[(1 + \alpha\beta^2)(\bar{y} - y^*)^2 + \frac{1}{1 + \alpha\beta^2} \cdot \sigma^2\right].$$

Since $1 + \alpha\beta^2 > 1$, the impact of $(\bar{y} - y^*)^2$ [gap between desired and potential GDP] is higher on EU_t^2 than on EU_t^1, which is due to the $CB's$ unsuccessful attempt to stimulate GDP beyond potential. As $\frac{1}{1+\alpha\beta^2} < 1$, the impact of shocks is lower on EU_t^2 than on EU_t^1, which is due to the $CB's$ stabilization response.