3 Achieving efficiency in overlapping generations models

1. Fiat money

Given the failure of the First Welfare Theorem in overlapping generations models, there is no guarantee that the consumption allocation of a general competitive equilibrium be Pareto efficient. This observation leads to the consideration of mechanisms allowing the economy to reach a Pareto efficient consumption allocation.

This section focuses on the mechanism called "fiat money", an intrinsically worthless asset accepted now in exchange for the good under the belief that it will be also accepted in exchange for the good in the future.

To illustrate the potential of fiat money to improve efficiency, consider the economy described Bhattacharya (2008); see also Champ et al. (2011, ch. 1). All generations are identical, each grows at a constant rate $n > 0$, and old people have no endowment. Specifically, for all generations t and s, $i \in N(t)$, and $j \in N(s)$: $N(t) = (1+n) \cdot N(t-1)$, $u_t^i = u_s^j$, and $w_t^i = w_s^j = (w, 0)$.

Since all young individuals are identical, inside money (loans) is not possible, because there is no loan market (all young individuals would like to lend and no one would like to borrow). Therefore, there is no trade and consumers must consume their endowments. The consequene of this autarkic state of the economy is that the old starve. The aim is to show that each generation's welfare can be maximized by resorting to fiat money.

The consumption allocation that maximizes generation t's welfare is obtained by maximizing the common utility function $u_t^i\left(c_t^i(t), c_t^i(t+1)\right)$ subject to the resource constraint in period t

$$N(t) \cdot c_t^i(t) + N(t-1) \cdot c_{t-1}^i(t) = N(t) \cdot w$$

where w is the young person's endowment. The right-hand side represents the total supply of the good available in period t, whereas the left-hand side stands for the total demand for the good in t, namely, the amount $N(t) \cdot c_t^i(t)$ that the young demand plus the amount $+N(t-1) \cdot c_{t-1}^i(t)$ that the old demand.

As $N(t) = (1+n) \cdot N(t-1) > 0$ and $c_{t-1}^i(t) = c_t^i(t+1)$,

$$c_t^i(t) + \frac{c_t^i(t+1)}{1+n} = w$$

where n can be viewed as a short of "biological interest rate".

For simplicity, let $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$ and assume that $c_t^i(t+1)$ represents the utility of the generation t individual that is old in $t+1$. Then the solution satisfies

$$1+n = \frac{c_t^i(t+1)}{c_t^i(t)}$$

$$c_t^i(t+1) = \left[w - c_t^i(t)\right] \cdot (1+n).$$

Consequently, $c_t^i(t) = \frac{w}{2}$ and $c_t^i(t+1) = (1+n) \cdot \frac{w}{2}$.

In autarky, utility for each young individual is $u_t^i(w, 0) = 0$; for each old individual, utility is $c_t^i(t+1) = w_t^i(t+1) = 0$ (without borriwing, no endowment means no consumption).

In the welfare-maximizing solution, every individual in t gets positive utility: when the individual is young, he obtains $u_t^i\left(\frac{w}{2}, (1+n) \cdot \frac{w}{2}\right) = \frac{(1+n)\cdot w^2}{4} > 0$; when old, he obtains $c_t^i(t+1) = \frac{(1+n)\cdot w}{2} > 0$.

The welfare-maximizing solution could be regarded as the one that a social planner would choose. Is this solution achievable through a money market?

Imagine that the old invent fiat money in period 1: a worthless asset intended to be generally accepted in exchange for the good in a competitive market.

Let M be the amount of fiat money created in $t = 1$ and, for all t, let $p(t)$ designate the price of the good in terms of money: in t, one unit of good is worth $p(t)$ units of money.

This suggests that $p(t)$ can be interpreted as the price level in the economy, whereas $1/p(t)$ would be the price, value or purchasing power of money (amount of the good that one unit of money can purchase in period t).

Thanks to the money market, a young individual could save purchasing power for the future by holding money. Let $m^i(t)$ represent the number of money units bought by an individual i who is young in period t. As a result, when young, i's budget constraint becomes

$$c_t^i(t) + \frac{m^i(t)}{p(t)} = w \tag{1}$$

and, when old, his budget constraint is

$$c_t^i(t+1) = \frac{m^i(t)}{p(t+1)}\,. \tag{2}$$

Money demand per person in t is $m^i(t) = p(t) \cdot \left[w - c_t^i(t)\right]$. Total money demand in t is then $N(t) \cdot m^i(t)$.

In equilibrium, total money demand in t equals total money supply in t. That is, $N(t) \cdot m^i(t) = M$. Consequently,

$$p(t) = \frac{M}{N(t) \cdot \left[w - c_t^i(t)\right]}\,.$$

This relationship is also valid for $t+1$:

$$p(t+1) = \frac{M}{N(t+1) \cdot \left[w - c_{t+1}^i(t+1)\right]}\,.$$

Given that all generations are identical, $c_{t+1}^i(t+1) = c_t^i(t)$. Thus, it follows from $N(t+1) = (1+n) \cdot N(t)$ that

$$\frac{p(t)}{p(t+1)} = \frac{N(t+1)}{N(t)} = 1 + n\,.$$

The above is the <u>equilibrium condition in the money market</u>. The ratio

$$P = \frac{p(t)}{p(t+1)}$$

is the <u>gross return of fiat money</u>: it is the amount of good earned in $t+1$ by investing in t one unit of good in money. One unit of the good in t can get (can be exchanged for) $p(t)$ units of money in t. As each unit of money in $t+1$ buys $1/p(t+1)$ units of the good in $t+1$, $p(t)$ can buy $P = p(t)/p(t+1)$ units of the good. All in all, one unit of the good invested in money in period t yields P units of good in period $t+1$. The following sketch summarizes the explanation.

$$1 \text{ unit of the good in } t \;\rightarrow\; p(t) \text{ units of money in } t \;\rightarrow\; \frac{p(t)}{p(t+1)} \text{ units of good in } t+1$$

The interpretation of n as a (net, real) interest rate can be taken to mean that the future (real) value of an asset is $(1+n)$ times the present (real) value of the asset. In the case at hand, the asset is money and the real value in period t of one unit of money is $\frac{1}{p(t)}$. This just follows from the fact that $\frac{1}{p(t)}$ measures the purchasing power in t of one unit of money.

With gross real interest rate assumed to be $1+n$, the real value in $t+1$ of $\frac{1}{p(t)}$ is $\frac{1}{p(t)} \cdot (1+n)$. This has to be the real value the asset "money" in $t+1$. By definition, the real value of money in $t+1$ is its purchasing power $\frac{1}{p(t+1)}$. Summarizing, it must be that

$$\frac{1}{p(t+1)} = \frac{1}{p(t)} \cdot (1+n)$$

or, rearranging,

$$\boxed{\frac{p(t)}{p(t+1)} = 1+n} \tag{3}$$

which is the equilibrium condition in the money market. This condition can then be understood as sustained by arbitrage: the return (in real terms) $1+n$ of making a hypothetical loan of the good should the same as the return $\frac{p(t)}{p(t+1)}$ of investing in money.

The demand for money by a young individual i is obtained by maximizing $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$ subject to (1) and (2). That is, i chooses $m^i(t)$ to

$$\text{maximize} \left(w - \frac{m^i(t)}{p(t)}\right) \cdot \frac{m^i(t)}{p(t+1)}$$

where, since the money market is assumed competitive, the prices $p(t)$ and $p(t+1)$ are taken as given. After equating to zero the derivative with respect to $m^i(t)$, real money demand is

$$\frac{m^i(t)}{p(t)} = \frac{w}{2}.$$

Consumption when the individual is young and when he is old is given by

$$c_t^i(t) = w - \frac{m^i(t)}{p(t)} = \frac{w}{2}$$

$$c_t^i(t+1) = \frac{m^i(t)}{p(t+1)} = \frac{w \cdot \frac{p(t)}{2}}{p(t+1)} = \frac{w \cdot (1+n)}{2}.$$

These results show that fiat money (i) generates the consumption levels that maximize welfare and (ii) improves upon the no trade situation. The effectivity of fiat money as a mechanism to improve efficiency depends on its ability to be a deposit of value, namely, that individuals believe that the money bought when young will be accepted when old in exchange for goods. If the young in t no longer expect that the young in $t+1$ will be willing to accept money, then the young in t will not accept money. In this case, money is actually absolutely worthless and turns out to have no economic function.

The above problem has been solved by direct substitution of (1) and (2) into the utility function. The alternative procedure is to maximize utility subject to the lifetime budget constraint obtained by putting together (1) and (2) so that $m^i(t)$ cancels out:

$$maximize_{\{c_t^i(t),\, c_t^i(t+1)\}} \;\; u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$$

$$subject\ to \;\; c_t^i(t) + \frac{p(t+1)}{p(t)} \cdot c_t^i(t+1) = w_t^i(t)\,.$$

By setting $R = 1 + n$, the money market equilibrium condition (3) implies that $\frac{p(t+1)}{p(t)} = \frac{1}{R}$. As consequence, the lifetime budget constraint is actually the familiar requirement $c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) + \frac{w_t^i(t+1)}{R(t)}$ in disguise.

As a final remark, define the inflation rate in period t as

$$\pi(t) = \frac{p(t) - p(t-1)}{1+n}.$$

Then

$$1 + \pi(t) = \frac{1}{1+n}$$

or, equivalently,

$$\boxed{\pi(t) = -\frac{n}{1+n}.}$$

This says that the economy experiences deflation at a constant rate. Since

$$\pi(t) = -\frac{1}{1+\frac{1}{n}}$$

deflation is more intense the rate n at which population grows:

$$\uparrow n \;\; \Rightarrow \downarrow \frac{1}{n} \Rightarrow \uparrow \frac{1}{1+\frac{1}{n}} \;\; \Rightarrow \uparrow |\pi| \, .$$

In addition, $c_t^i(t+1) = \frac{w/2}{1+\pi(t+1)}$: the old consume half of the (inflation-based) present value of the young's endowment.

2. Pensions

This section presents a superficial analysis of pensions; deeper and more detailed analyses can be found, for instance, in Wickens (2008, pp. 138-143) and Heijdra (2009, pp. 624-647).

Consider an economy in which individuals initially can only consume or lend/borrow in the loan market. A government that plans to introduce a pension system has two basic options.

• Adopt a fully funded pension system. In this system, the government taxes the young in t, lends the revenues, and pays out the proceeds to the old in the next period $t+1$ as a pension.

• Establish an unfunded or pay-as-you-go pension system. What characterizes this system is that the pension $p(t)$ to the old in period t are paid out from current tax receipts $\tau(t)$ on the young.

Under fully funded pensions, a young individual pays his own pension through the government: the government takes savings from young individuals, invests them, and, when the individuals become old, are paid the pension from the revenues of the investment. This procedure suggests the following question: are the young people forced to save more than they wish? In fact, when young, i's budget constraint is

$$c_t^i(t) + l^i(t) + \tau(t) = w_t^i(t);$$

when old, it is

$$c_t^i(t+1) = w_t^i(t+1) + R(t) \cdot \left[l^i(t) + \tau(t)\right]$$

where l^i represents what i voluntary saves and τ is the tax paid to the government. The government invests τ to get the market return R and, in the next period, transfer the proceeds $R \cdot \tau$ to the individual.

Inspection of the budget constraints indicates that the pension has no effect on the savings decision since budget constraints formally coincide with the constraints without the pension: $l^i + \tau$ simply replaces l^i. Voluntary savings are cut to pay taxes so that income remains the same. More specifically, if i chooses to save l^i without pension, then, under the fully-funded pension system with tax τ, the individual saves $l^i - \tau$. Even though the government manages τ, in the end, the individual receives the same proceeds as if he managed τ.

The interesting case is then the PAYGO system: the pension $p(t)$ assigned to the old in t comes from the taxes $\tau(t)$ currently paid by a different generation, the young people. To illustrate the implications of the system, suppose population grows at rate n. Assuming that the government chooses to balance the budget, the government budget constraint in period t is

$$\tau(t) \cdot N(t) = p(t) \cdot N(t-1).$$

In words, the total amount $\tau(t) \cdot N(t)$ of tax receipts in t from the generation born in t must equal the total amount $p(t) \cdot N(t-1)$ of pensions paid in t to the generation that is old in t (hence, the generation born in $t-1$). As population grows at a constant rate n,

$$\tau(t) \cdot (1+n) \cdot N(t-1) = p(t) \cdot N(t-1)$$

and, therefore,

$$\tau(t) \cdot (1+n) = p(t).$$

When young, i's budget constraint is

$$c_t^i(t) + l^i(t) + \tau(t) = w_t^i(t);$$

when old, it is

$$c_t^i(t+1) = w_t^i(t+1) + R(t) \cdot l^i(t) + p(t) = w_t^i(t+1) + R(t) \cdot l^i(t) + \tau(t) \cdot (1+n).$$

His lifetime budget constraint is

$$c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) + \frac{w_t^i(t+1)}{R(t)} + \tau(t) \cdot \left(\frac{1+n}{1+r(t)} - 1\right).$$

Without the pension, the term $\tau(t) \cdot \left(\frac{n-r}{1+r(t)}\right)$ is missing. If $n > r(t)$, then the budget set with the pension is larger, so a more preferred consumption basket is feasible.

If $n < r(t)$, then the budget set with the pension is smaller. As the welfare maximizing basket without the pension is not feasible now, the pension reduces the young's welfare.

The whole system is like a pyramid scheme: a club is created so that new members must pay to existing members and will be paid by future members. Loosely speaking, what a new member pays is captured by r, whereas the availability of future members is determined by n. As long as the pool of potential members is larger than the club of existing members, it is profitable to join the club.

In an economy with an expanding population, the intuition is that there are enough young people to finance the pensions of the current old. The precise condition is actually $n > r$. For it is not 1 that has to be transferred to an old individual, but $1 + r$: for every unit of good that the old did contribute as young, he is entitled to receive $1 + r$. Hence, if each young individual pays 1, a total of $1 + n$ individuals is required.

For instance, suppose $r = 1/2$ and that there are 100 old individuals. If the taxes each old individual paid when young is normalized to 1, then the collective of old individuals is entitled to receive pensions worth $100 \cdot \left(1 + \frac{1}{2}\right) = 150$. If the number of currently young individuals remained at 100 (indicating that $n = 0$), then, if taxes do not change, the government can only collect 100 from the young people. This amount is insufficient to cover the need of 150. Accordingly, n must at least be such that $100 \cdot (1 + n) = 150$. That is, $n = 1/2$ is the smallest value that makes the system viable when $r = 1/2$ and taxes do not change.

If taxes can change, then to sustain a PAYGO system when $n < r$, the young must be over-taxed with respect to previous generations. In the end, what matters is that, collectivelly, the young can finance the old. This can be done by increasing the number of tax-payers and/or by increasing what each tax-payer contributes.

3. Storage technology

This section introduces a technology that allows the good to be stored from one period to the next; for more on the analysis of this possibility, see McCandless and Wallace (1991, ch. 8). To illustrate the effects of this kind of technology, consider the following economy.

The utility function is $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$ for each individual i. Every generation is made up of two grups, 1 and 2. There are n_i members in group $i \in \{1, 2\}$. Each member of grup 1 has endowment $(0,\ w_1')$. Each member of grup 2 has endowment $(w_2,\ w_2')$. There is a free technology allowing individuals to transfer (store) good from one period to the next: for each unit of the good that an individual accumulates in period t, the same individual receives λ units of the good in $t+1$, where $0 < \lambda \leq 1$. The value $1 - \lambda$ can be seen as the cost of using the technology because this is the amount of good lost when one unit of the good is stored. It can also be viewed as a measure of the depreciation of the good.

It is the presumption that the technology is simply a storage technology that forces the constraint $\lambda \leq 1$: one cannot get in $t+1$ more good than has been accumulated in t. The models that have been considered so far could be associated with the case $\lambda = 0$. The case $\lambda > 1$ represents a production technology and will be dealt with in subsequent lessons.

For the purpose of comparison, consider first $\lambda = 0$ (with just a loan market). In this case, the savings function of a member of group 1 is $s_1 = -\frac{w_1'}{2 \cdot R}$ and the savings function of a member of group 2 is $s_2 = \frac{w_2}{2} - \frac{w_2'}{2 \cdot R}$. The total savings function is $S = n_1 \cdot s_1 + n_2 \cdot s_2 = \frac{n_2 \cdot w_2}{2} - \frac{n_1 \cdot w_1' + n_2 \cdot w_2'}{2 \cdot R}$. In equilibrium, $S = 0$. As a consequence,

$$R = \frac{n_1 \cdot w_1' + n_2 \cdot w_2'}{n_2 \cdot w_2} = \frac{\frac{n_1}{n_2} \cdot w_1' + w_2'}{w_2}.$$

If $R > 1$, then the introduction of a storage technology (which implies $\lambda \leq 1$) would be useless. By lending in t one unit of the good, one receives $R > 1$ in $t+1$. Hence, there is no need to resort to a technology that gives you λ units in $t+1$ in exchange for one unit in t. For example, if $w_1{}' = 8$, $w_2 = 6$, $w_2{}' = 2$, and $n_1 = n_2$, then $R = 5/3$ and a storage technology is useless in this case.

This points to an important general conclusion: <u>if a storage technology allows every unit in t to become $\lambda \leq 1$ units in $t + 1$, then the interest rate $R(t)$ must be at least λ</u>. The reason is simple: by storing one unit today, you get λ tomorrow; therefore, by lending one unit today, you cannot be repaid less than λ tomorrow, because in that case you would prefer storing to lending. In the case $\lambda = 0$ covered so far, this constraint amounted to $R \geq 0$: having $R < 0$ means that you relinquish good today by lending and that you also must give up good tomorrow (you lend today and you have to pay for it tomorrow).

Suppose now that $0 < \lambda \leq 1$. For young individual i of generation t, let $k^i(t + 1)$ designate the amount of good that i accumulates (his "capital"). For any individual of group 1, the budget constraints (when young and old, respectively) are, in simplified notation,

$$c_1 + k_1' + l_1 = 0 \quad \text{and} \quad c_1' = w_1' + \lambda \cdot k_1' + R \cdot l_1.$$

Dividing by R the second equation and adding up both, the lifetime budget constraint of a member of group 1 is

$$c_1 + \frac{c_1'}{R} = \frac{w_1'}{R} + k_1' \cdot \left(\frac{\lambda}{R} - 1\right).$$

By the general conclusion above, $R \geq \lambda$: <u>if $R < \lambda$, then the loan market ceases to exist</u>. On the other hand, <u>if $R > \lambda$, then the storage technology is useless</u>: it is as if $\lambda = 0$. In view of this, $R > \lambda$ leads to the solution for the case in which there is no storage technology. This leaves the case $R = \lambda$ as the only one in which the loan market can coexist with a storage technology that can actually be used.

The lifetime budget constraint of a member of group 2 is

$$c_2 + \frac{c_2'}{R} = w_2 + \frac{w_1'}{R} + k_1' \cdot \left(\frac{\lambda}{R} - 1\right).$$

If attention is restricted to the case $R = \lambda$, then the lifetime budget constraints are

$$c_1 + \frac{c_1'}{R} = \frac{w_1'}{R} \quad \text{and} \quad c_2 + \frac{c_2'}{R} = w_2 + \frac{w_1'}{R}.$$

These are the same constraints that arise when the technology does not exist, for which reason the aggregate savings function is given by $S = \frac{n_2 \cdot w_2}{2} - \frac{n_1 \cdot w_1' + n_2 \cdot w_2'}{2 \cdot R}$, as previously calculated. The important difference is that, in equilibrium, savings must finance investment: the capital accumulated by individuals should be considered a form of investment. Summing up, the new equilibrium condition is $S = K'$, where $K' = n_1 \cdot k_1' + n_2 \cdot k_2'$.

For instance, let $w_1{}' = 8$, $w_2 = 6$, $w_2{}' = 2$, and $n_2 = 4 \cdot n_1$. It then follows that $S = n_1 \left(12 - \frac{8}{R}\right)$. Without the storage technology, in equibrium, $S = 0$, so $R = \frac{2}{3}$. With the technology, in equilibrium, $n_1 \left(12 - \frac{8}{R}\right) = n_1 \cdot k_1{}' + 4 \cdot n_1 \cdot k_2{}'$; that is, since $R = \lambda$ has been assumed,

$$k_1{}' = 12 - \frac{8}{\lambda} - 4 \cdot k_2{}' \cdot \qquad (4)$$

Condition (4) states the relationship between the amount of stored good by the two groups that must hold in order to have a general equilibrium in which $R = \lambda$, namely, individuals are indifferent between lending or storing the good. Since there are many pairs $(k_1{}', k_2{}')$ satisfying (4), the final conclusion is that there are infinitely many equilibria in the economy (because now a general competitive equilibrium must specify the amount of capital accumulated by every individual in every period).

For example, with $\lambda = 4/5$, (4) becomes $k_1{}' = 2 - 4 \cdot k_2{}'$, where $k_1{}' \geq 0$ and $k_2{}' \geq 0$. Accordingly, if $k_2{}' = \frac{1}{4}$, then $k_1{}' = 1$; if if $k_2{}' = 0$, then $k_1{}' = 2$; and if $k_2{}' = \frac{1}{2}$, then $k_1{}' = 0$.

To verify that $k_1{}' = 2 - 4 \cdot k_2{}'$ must hold in equilibrium, recall that $s_1 = -\frac{w_1'}{2 \cdot R} = -5$ and that $s_2 = \frac{w_2}{2} - \frac{w_2'}{2 \cdot R} = \frac{7}{4}$. Consequently,

$$S = S_1 + S_2 = -5 \cdot n_1 + 4 \cdot n_1 \cdot \frac{7}{4} = 2 \cdot n_1.$$

This is the total amount of savings in the economy, given $R = \lambda = 4/5$. Hence, $2 \cdot n_1$ must represent the total amount K' of good stored in every period: $K' = n_1 \cdot k_1{}' + n_2 \cdot k_2{}' = n_1 \cdot (k_1{}' + 4 \cdot k_2{}')$. For this reason, $S = K'$ amounts to

$$2 \cdot n_1 = n_1 \cdot (k_1{}' + 4 \cdot k_2{}')$$

or, precisely,

$$2 = k_1{}' + 4 \cdot k_2{}'$$

that is,

$$k_1{}' = 2 - 4 \cdot k_2{}'.$$

4. Exercises

Exercici 1. Diner fiduciari en expansió i població constant. Considera el model d'una economia monetària descrit en la secció 1 amb l'única diferència que la població no creix ($n = 0$) i la quantitat de diner creix cap període a una taxa constant $m > 0$. Calcula el valor de la taxa d'inflació i, en concret, el seu signe.

Exercici 2. Diner fiduciari i població en expansió. Considera el model d'una economia monetària descrit en la secció 1 amb l'única diferència que la quantitat de diner creix cap període a una taxa constant $m > 0$. Expressa $\frac{p(t)}{p(t+1)}$ i la taxa d'inflació $\pi(t)$ en funció d'n i d'm.

Exercici 3. Teoria quantitativa del diner. En el model descrit en la secció 1, és certa la implicació de la teoria quantitativa del diner segons la qual el nivell de preus és proporcional a l'estoc de diner? En concret, que passaria amb el nivell de preus en un determinat periode t si l'estock de diner és dupliqués?

Exercici 4. Equilibri amb diner i préstecs. La funció d'utilitat de cada consumidor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació està formada per 100 individus, 60 amb dotació $(1, 0)$ i els altres 40 amb dotació $(0, 1)$.

(i) Calcula l'equilibri general si només hi ha un mercat de préstecs.

(ii) Calcula l'equilibri general si, a més del mercat de préstecs (del bé), hi ha un mercat de diner on es compra i ven una unitat (perfectament divisible) de diner fiduciari creada i venuda en el període inicial per la població de gent gran. Determina la funció de demanda de diner dels joves. Compara la utilitat que cada individu obté en aquest i en l'anterior apartat.

Exercici 5. Equilibri amb pensions. La funció d'utilitat de cada consu-midor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació está formada per 40 individus amb dotació $(0, 2)$ i 60 amb dotació $(1, 0)$.

(i) Calcula l'equilibri general si només hi ha un mercat de préstecs.

(ii) El govern estableix un sistema de pensions de repartiment finançat amb un impost de 0,6 a pagar per cada jove amb dotació positiva quan és jove. La recaptació de l'impost a cada període es distribueix igualitàriament entre tots els individus grans del període. Calcula l'equilibri general.

(iii) El govern estableix un sistema de pensions de repartiment finançat amb un impost de τ unitats a pagar per cada jove amb dotació positiva quan és jove. La recaptació de l'impost a cada període es distribueix igualitàriament entre tots els individus grans del període. Calcula el valor de τ que fa que, en l'equilibri general, el consum de cada individu sigui el mateix valor c, tant si és jove com si és gran.

Exercici 6. Finançament col·lectiu de tecnologia d'emmagatzematge. La funció d'utilitat de cada consumidor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació está formada per dos grups, 1 i 2, cadascun format per n individus. Cada membre d'un grup 1 té dotació $(0, w)$ i cada membre del grup 2 compta amb dotació $(0, v)$, on $w > v > 0$. La naturalesa del bé no permet de transferir-lo d'un període a cap altre posterior.

Existeix, però, la possibilitat de crear una tecnologia d'emmagatzematge del bé durant un període. Gràcies a aquesta tecnologia, per cada unitat del bé acumulada en el període t per un individu i, restarà disponible per a i en el període $t+1$ la quantitat $\lambda(t)$ del bé, on $0 < \lambda(t) < 1$.

L'efectivitat d'aquesta tecnologia depèn de les contribucions al seu desenvolupament que facin els individus. Si, en t, cada individu del grup 1 aporta τ_1 unitats del bé per a finançar/desenvolupar la tecnologia i cada i cada individu del grup 2 aporta τ_2 unitats, aleshores $\lambda(t) = \frac{\tau_1+\tau_2}{w+v}$. Determina, en l'equilibri general, quina part de la seva dotació estalvia, i quina aporta al finançament de la tecnologia, cada individu.

Exercici 7. Tecnologia de transferència. Considera una economia on tots els individus són iguals, viuen durant dos períodes consecutius i el bé només pot existir durant un període. Imagina que es descobreix una tecnologia que, sense cost, permet de transferir una unitat del bé dos períodes cap al futur. Així, si un individu acumula una unitat del bé en el període t fent servir la tecnologia, aquesta unitat estarà disponible per a ser consumida (o novament acumulada) en el període $t+2$. Tindria aquesta tecnologia utilitat pràctica? En particular, acumularien bé els individus? [Opcional: formula un model on quedi representada aquesta tecnologia i determina l'equilibri general].

Exercici 8. Equilibri amb tecnologia d'emmagatzematge imperfecta. La funció d'utilitat de cada consumidor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació está formada per dos grups, 1 i 2, cadascun format per n individus. Cada membre d'un grup 1 té dotació (v, w) i cada membre del grup 2 compta amb dotació (w, v), on $w > v > 0$. Tot i que la naturalesa del bé no permet de transferir-lo d'un període a cap altre posterior, existeix una tecnologia que possibilita l'acumulació del bé: per cada unitat del bé que un individu jove acumuli en el període t, l'individu disposarà de $0 < \lambda < 1$ unitats del bé en el període $t+1$. Assumint que hi ha un mercat de préstecs del bé, calcula l'equilibri general.

5. References

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