4. Exchange rate

1. The exchange rate

Definition 1.1. The nominal exchange rate e (or, for short, exchange rate) between two currencies is the price of one currency in terms of the other.

The exchange rate allows domestic purchasing power to be spent abroad. The exchange rate is the basic macroeconomic variable connecting economies with different currencies.

Example 1.2. If $e = 2$ \$/€, one euro can be traded for two dollars: the price in dollars of one euro is two dollars. The inverse $e' = 1/2$ \$/€ of $e = 2$ \$/€ shows how many euros can be traded for one dollar: the price in euros of one dollar is 0.5 euros. Accordingly, both e and e' express the same information.

2. Quoting an exchange rate

Definition 2.1. The direct quotation of an exchange rate expresses the exchange rate as domestic (home) currency units / foreign currency units.

Definition 2.2. The indirect quotation of an exchange rate expresses the exchange rate as foreign currency units / domestic (home) currency units.

Example 2.3. If the euro is the home currency, then $e = 2$ \$/€ quotes the exchange rate indirectly. When the peseta was the Spanish currency, direct quotation was the norm: $e = 150$ Pts/\$.

Remark 2.4. The quotation chosen determines the units of e.

Direct quotation is the "natural" way of quoting an exchange rate. The domestic price of a commodity is typically expressed as domestic currency units per commodity unit; for instance, 1.2 € per kilogram of flour. Considering the foreign currency as another commodity, the price of the foreign currency would then be expressed as domestic currency units per foreign currency unit. Despite this, indirect quotation is more convenient because an increase in the value of the domestic currency (with respect to the foreign currency) is represented by a rise in the exchange rate when quoted indirectly, whereas it is represented by a fall when quoted directly.

3. Currency appreciation

Definition 3.1. A currency X appreciates with respect to another currency Y if the number of units of Y that one unit of X can buy increases.

When X appreciates with respect to Y, currency X becomes more valuable in terms of Y. When indirect quotation is used, the home currency appreciates when the exchange rate rises. Under direct quotation, the home currency appreciates when the exchange rate falls.

Example 3.2. In passing from $e = 1$ \$/€ to $e' = 2$ \$/€, the euro appreciates with respect to the dollar. Initially, one euro could be traded for only one dollar; after the exchange rate jump, one euro can be traded for two dollars, for which reason the euro has increased its value.

Example 3.3. In passing from $e = 2$ €/¥ to $e' = 1$ €/¥, the euro appreciates with respect to the yen. Initially, two euros were needed to buy one yen; after the fall of the exchange rate, only one euro is required to buy one yen, so the euro has increased its value

4. Currency depreciation

Definition 4.1. A currency X depreciates with respect to another currency Y if the number of units of Y that one unit of X can buy diminishes.

When X depreciates with respect to Y, currency X becomes less valuable in terms of Y. When indirect quotation is used, the home currency depreciates when the exchange rate falls. Under direct quotation, the home currency depreciates when the exchange rate rises.

Example 4.2. In passing from $e = 2$ \$/€ to $e' = 1$ \$/€, the euro depreciates with respect to the dollar. Initially, one euro could be traded for two dollars; after the rise in the exchange rate, one euro can only be traded for one dollar and, accordingly, the euro has reduced its value.

Example 4.3. In passing from $e = 1$ €/¥ to $e' = 2$ €/¥, the euro depreciates with respect to the yen. Initially, one euro could buy one yen; after the exchange rate falls, one euro can only buy 0.5 yen and, therefore, the euro has lost value.

5. The currency (or foreign exchange) market

http://en.wikipedia.org/wiki/Foreign_exchange_market

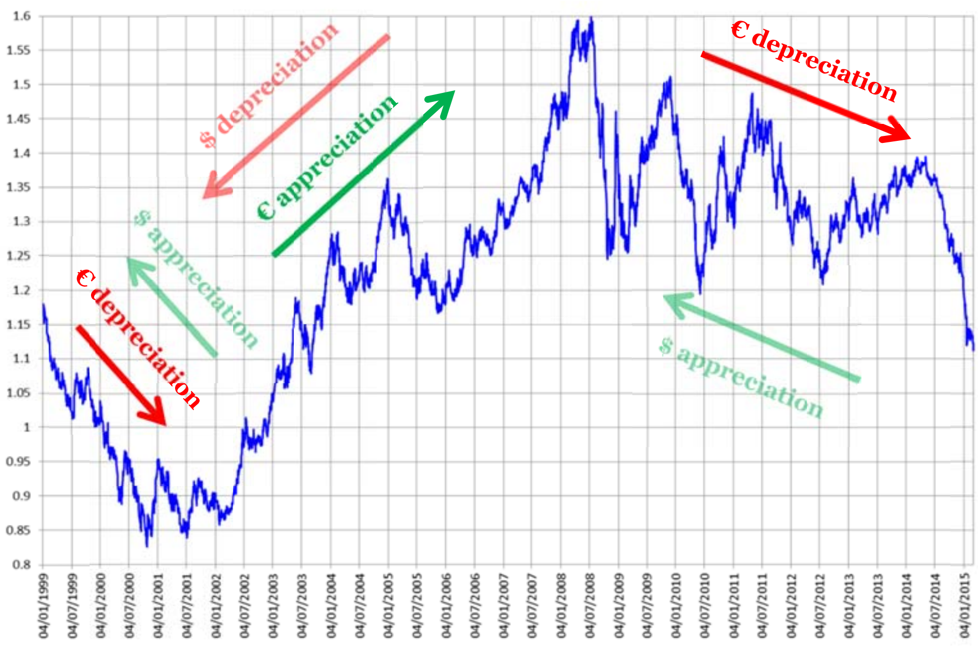

Fig. 1. Exchange rate \$/€, 4 January 1999 - 4 March 2015

Definition 5.1. The currency market is the market for the trading of currencies.

The currency market is the largest and more liquid financial market in the world. Average trading in currency markets in April 2013 was \$5.3 trillion per day (\$4.0 trillion in April 2010; \$3.3 trillion in April 2007). 70% to 90% of all the transacctions are speculative. The main traders are banks. Interbank trading accounts for more of the 50% of all transactions.

In May 2014, the top 10 currency traders were: Citi (market share, 16%), Deutsche Bank (15.6%), Barclays Investment Bank (10.9%), UBS AG (10.8%), HSBC (7.1%), JP Morgan (5.5%), Bank of America Merill Lynch (4.3%), Royal Bank of Scotland (3.2%), BNP Paribas (3.1%), and Goldman Sachs (2.5%). By value, the most traded currencies are the dollar (87% of daily share in April 2013), the euro (33.4%), the Japanese yen (23%), the pound sterling (11.8%), the Australian dollar (8.6%), and the Swiss franc (5.2%).

	USD	GBP	CAD	EUR	AUD
	1	0.59676	1.10661	0.72389	1.11928
	1.67571	1	1.85436	1.21302	1.87559
	0.90366	0.53927	1	0.65415	1.01145
	1.38143	0.82439	1.52871	1	1.54621
	0.89343	0.53317	0.98868	0.64674	1

Refresh in 0:33 | Feb 28, 2014 17:28 UTC

	USD	GBP	CAD	EUR	AUD
	1	0.67704	1.27736	0.94978	1.30688
	1.47701	1	1.88668	1.40283	1.93028
	0.78286	0.53003	1	0.74355	1.02311
	1.05288	0.71285	1.34491	1	1.37599
	0.76518	0.51806	0.97741	0.72675	1

Refresh in 0:53 | Mar 16, 2015 06:45 UTC

Fig. 2. Exchange rates, 28 Feb 2014 and 16 Mar 2015 (€1 exchanges for $1.05288) | http://www.x-rates.com/

Example 5.2. Fig. 2 indicates that the euro depreciated against the dollar from the 28 Feb 2014 to the 16 Mar 2015 (the exchange rate decreased from 1.38 \$/€ to 1.05 \$/€). In the same period, the British pound appreciated against the Canadian dollar (the exchange rate increased from 1.85 C\$/£ to 1.88 C\$/£).

6. The currency market model: a non-standard version

Imagine all the suppliers of euros, who sell them in exchange for dollars. Each supplier is supposed to be represented by an offer (x, y) of the sort "give me y dollars and I will give you x euros". Arrange the suppliers from smaller to larger ratio y/x. The y/x ratio corresponding to offer (x, y) expresses the supplier's exchange rate: the rate at which the supplier is willing to exchange euros for dollars.

It is then possible to define an increasing function EU (since suppliers of euros are typically eurozone citizens) that, for $n = 1, 2, \dots$, associates with the amount $q_€$ of euros supplied by the first n suppliers in the ordering, the corresponding total amount $q_\$$ of dollars that these n suppliers ask in return. Fig. 3 represents this function when many suppliers exist. At a point like a (European) suppliers offer $q_€^a$ euros in exchange for $q_\$^a$ dollars. The convexity of the function follows from the assumption that suppliers making offers with smaller exchanger rate are represented first in the EU function.

Example 6.1. There are four euro suppliers, A, B, C, and D. Their offers are, respectively, (15, 60), (10, 10), (20, 30), and (5, 15). The associated exchange rates are 4, 1, 1.5, and 3. The ordering of suppliers is B, C, D, and A. The EU function f would be $f(10) = 10$ (offer by supplier B), $f(30) = 40$ (offer by suppliers B and C), $f(35) = 55$ (offer by suppliers B, C, and D), and $f(50) = 105$ (offer by all four suppliers).

The same analysis would apply to the suppliers of dollars in exchange for euros. An increasing function US analogous to the function EU could be defined (since suppliers of dollars are typically US citizens, the function is labelled "US"). Fig. 4 depicts the function US; at a point like a (American) suppliers offer $q_\$^a$ dollars in exchange for $q_€^a$ euros. This function would be convex when drawn in the plane in which the horizontal axis (the x-axis) measures the amount of dollars and the vertical axis (the y-axis) measures the amount of dollars. When the axes are inverted, to represent simultaneously the US and EU functions, an originally convex function turns concave, as shown in Fig. 5.

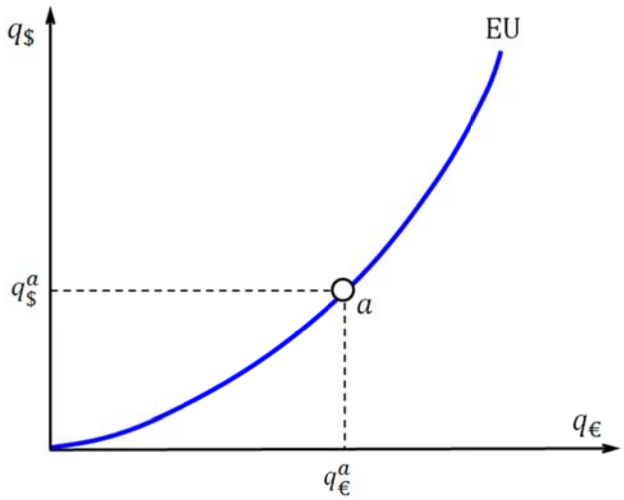

Fig. 3. Suppliers of euros that demand dollars

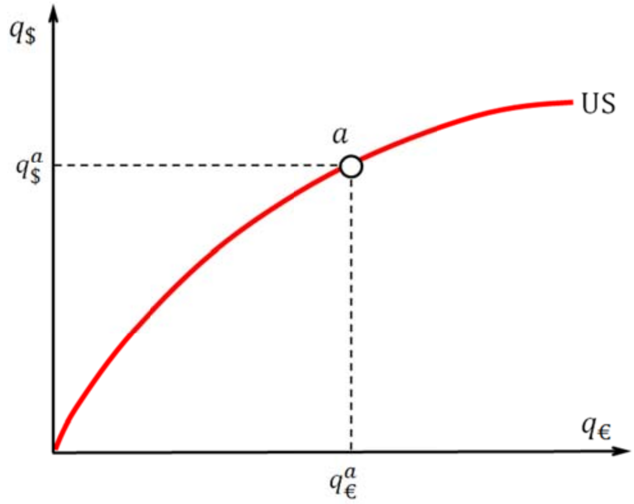

Fig. 4. Suppliers of dollars that demand euros

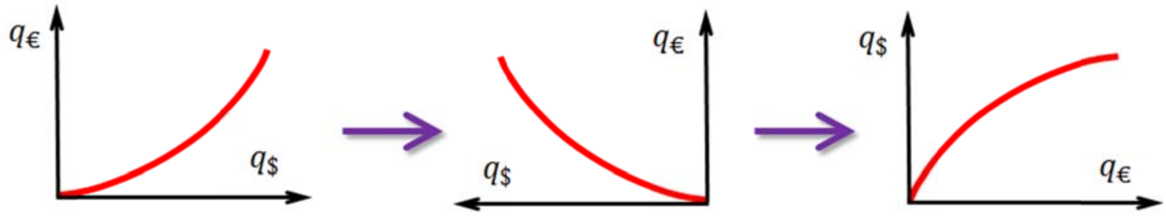

Fig. 5. How a convex funcion looks like concave when the labels of the axes are inverted

Fig. 6 puts the functions EU and US together. The model will be used to predict that <u>point g defines the volume of euros and dollars exchanged</u>: $q_€^*$ euros are exchanged for $q_\$^*$ dollars, which means that the exchange rate is the quotient $q_\$^*/q_€^*$. Geometrically, this rate is given by the slope of the line e^* connecting the origin with point g.

To justify the claim that g represents the volume of currencies traded, suppose otherwise: the volume traded of euros is different from $q_€^*$. The case in which that volume is smaller than $q_€^*$ in represented in Fig. 6, whereas the case in which it is larger is dealt with in Fig. 7.

Starting with Fig. 6, suppose American traders ask for only $q_€^0$ euros. Point a would represent the American traders' offer. At that point, they say: "we want $q_€^0$ euros and will pay $q_\$^0$ dollars for them". If point a represented the volume of currencies traded, then only the European traders offering the amount $q_€^0$ of euros will get dollars. Those traders will be willing to deliver $q_€^0$ euros in return for $q_\$^0$ dollars because, according to function EU, they demand much less in return: $q_\$^z$ euros (point z). But, if $q_\$^0$ dollars are going to be paid in exchange for euros, more European traders would be willing to trade: function EU establishes that, in return for $q_\$^0$ dollars, there are European traders ready to offer $q_€^1$ euros, with $q_€^1 > q_€^0$. Since point a would lead to b, point a is not stable and, therefore, cannot be deemed a good prediction of the amount of currencies traded.

Yet, something similar occurs at point b. That point b is like European traders voicing "we want $q_\$^0$ dollars and will pay $q_€^1$ euros for them". If point b established the volume of currencies traded, then only the American suppliers offering the amount $q_\$^0$ of dollars will get euros. Those suppliers will be willing to deliver $q_\$^0$ dollars in return for $q_€^1$ euros since, according to function US, they demand much less in return: $q_€^0$ euros (point a).

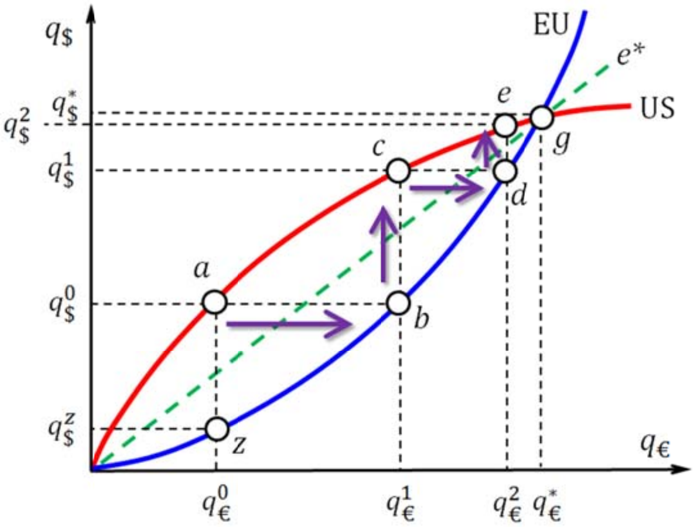

Fig. 6. Convergence to point g from the left

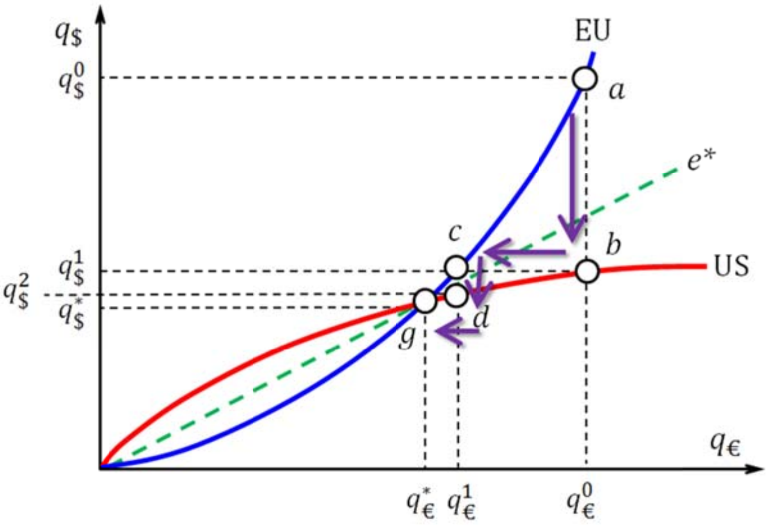

Fig. 7. Convergence to point g from the right

Having only $q_{€}^{1}$ euros exchanged for $q_{\$}^{0}$ dollars would leave some American traders insatisfied, as more traders than those offering just $q_{\$}^{0}$ dollars would like to trade. Specifically, the traders offering the difference $q_{\$}^{1} - q_{\$}^{0}$ would enter the market: as indicated by the US function, when they are offered $q_{€}^{1}$ euros, American traders would be willing to chase them in return for $q_{\$}^{1}$ dollars (point c). As a result, neither b could considered a stable state (or resting point) of the market: new American traders will join the existing traders present at b, so that the market would move from b to c.

Point c reproduces the situation of point a, for which reason the conclusion is that more European traders will enter the market and move it from point c to point d. Since the market situation represented by d is analogous to that represented by b, the market will move from d to e. And so on. This dynamics will eventually converge to point g.

Fig. 7 handles the other possibility: what if Americans asked for more than $q_{€}^{*}$ euros? Suppose they request $q_{€}^{0} > q_{€}^{*}$. As shown by the US function, American traders are only willing to pay $q_{\$}^{1}$ dollars in exchange for $q_{€}^{0}$ euros. Yet, according to the EU function, European traders only offer $q_{€}^{1}$ euros (with $q_{€}^{1} < q_{€}^{0}$) in return for $q_{\$}^{1}$ dollars (this is point c in the EU function). But to get the $q_{€}^{1}$ euros that European traders offer, American traders (point d) merely supply $q_{\$}^{2}$ euros (with $q_{\$}^{2} < q_{\$}^{1}$). This adjustment process will eventually reach point g.

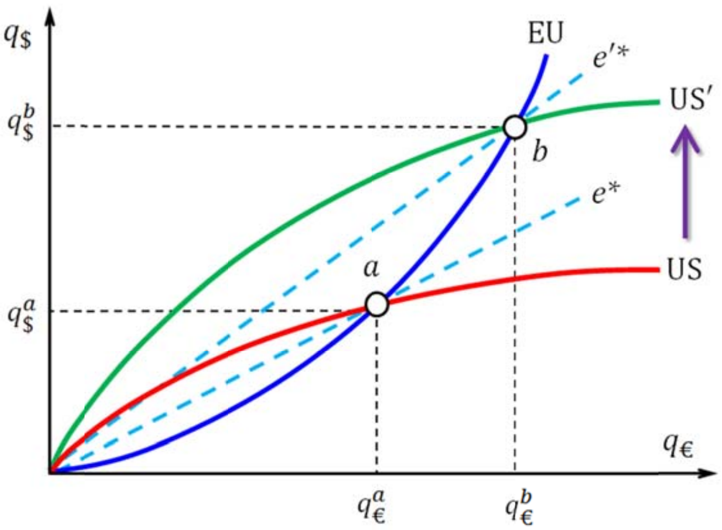

Fig. 8. American traders will pay more to get euros

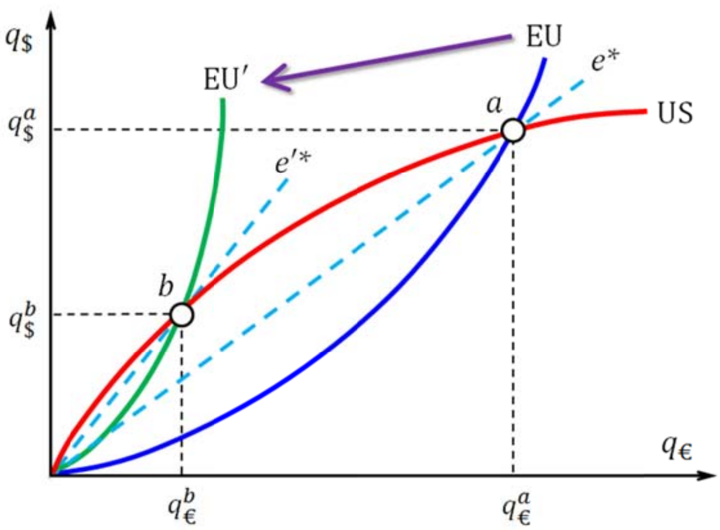

Fig. 9. European traders will pay less to get dollars

Figs. 8 and 9 put the model in action. Fig. 8 shows the effects of American traders willing to pay more dollars than before in exchange for euros. This change in the Americans' willingness to pay is represented in Fig. 8 by a shift to the left (or upwards) of the US function. If the market is initially at a, it eventually moves to b.

The consequences of having American traders ready to give more dollars in return for euros are: (i) the volume of trade increases (more dollars and more euros exchanged); and (ii) the market exchange rate (represented by the slope of the line joining the origin with the point representing the market resting state) increases (the slope of the line e'^* is higher than the slope of the line e^*).

Fig. 9 indicates the effects of European traders willing to offer fewer euros than before in exchange for dollars. This is represented by a shift of the EU function to the left. The passage from a to b implies: (i) the volume of trade decreases (for instance, the amount of euros exchanged goes from $q_€^a$ to $q_€^b < q_€^a$); and (ii) the market exchange rate increases (the slope of the line e'^* is higher than the slope of the line e^*).

Figs. 8 and 9 suggest that the two basic sources for an increase in the exchange rate $/€ are that American traders find the euro more attractive (which encourages them to give more dollars in return for euros) and that European traders find the dollar less appealing (which induces them to give fewer euros in return for dollars). The opposite events would lead to a decrease in the exchange rate.

These results can also be obtained in an equivalent model formulated using the more familiar tools of supply and demand functions. This is the model described next and that will be adopted to represent and analyze the currency market.

7. The currency market model: the standard version

In the standard version, the currency market is modelled as a competitive market. As in Section 6, for simplicity, only two currencies will be considered: the home and the foreign currency. The default interpretation will be that the euro is the home currency and the dollar is the foreign currency. As in the liquidity market model, it will be assumed that: (i) the supply of euros function $S_€$ slopes upwards; (ii) the demand for euros function $D_€$ slopes downwards; and (iii) both functions intersect at only one point.

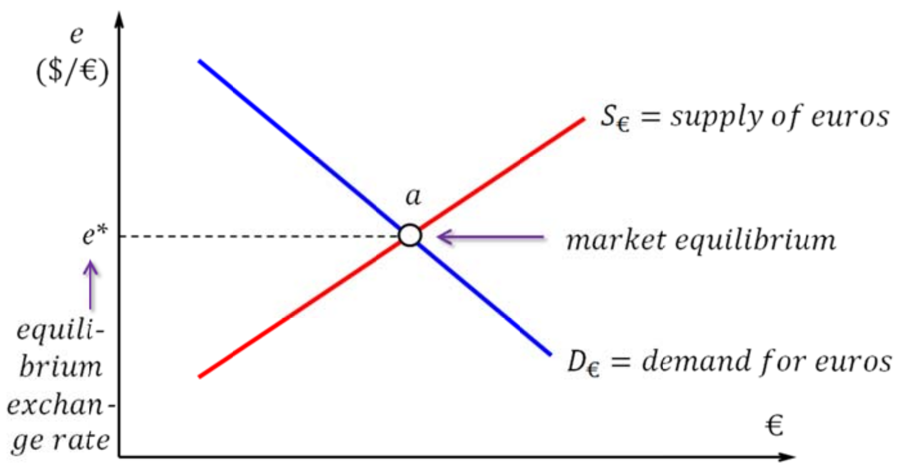

Fig. 10. Graphical representation of the currency market model

Fig. 10 represents graphically the currency market model, where quantity is the quantity of euros and price is the exchange rate $/€ quoted indirectly.

Definition 7.1. The market equilibrium of the currency market model is the pair (e^*, $q_€^*$) such that, when the exchange rate is e^*, the supply of euros is $q_€^*$ and the demand for euros is also $q_€^*$. The value e^* is the equilibrium exchange rate.

8. Demand for euros

The demand for euros is, at the same time, supply of dollars. The agents demanding euros have dollars but want to buy European goods and/or financial assets.

The demand for euros function slopes downward because a reduction in e means that fewer dollars are needed to purchase an euro. This makes European goods and financial assets comparatively cheaper. To buy more such goods and assets, so more euros are demanded. The following sketch summarizes the argument leading from ↓e to ↑quantity demanded of euros:

> ↓e ⇒ fewer dollars required to buy one euro ⇒ ↓price in dollars of European goods and financial assets ⇒ ↑quantity demanded by Americans of European goods and financial assets ⇒ ↑quantity demanded by Americans of euros (to buy the additional European goods and financial assets)

9. Supply of euros

The supply of euros is, at the same time, demand for dollars. The agents supplying euros want dollars to buy American goods and/or financial assets.

The supply function slopes upward because a rise in e means that more dollars are given in exchange for one euro, making American goods and financial assets comparatively cheaper. To buy more such goods and assets, more dollars are needed, so more euros are supplied. The following sketch summarizes the argument leading from ↑e to ↑quantity demanded of euros:

> ↑e ⇒ more dollars received for one euro ⇒ ↓price in euros of American goods and financial assets ⇒ ↑quantity demanded by Europeans of American goods and financial assets ⇒ ↑quantity supplied by Europeans of euros (to buy the additional American goods and financial assets)

10. The currency market model: some examples

Example 10.1. Effect on the equilibrium exchange rate of a rise in the European GDP; see Fig. 11. A rise in the European GDP means that Europeans have more income to spend. It is likely than spending on goods will increase. In particular, Europeans will spend more on American goods. To buy the additional American goods, Europeans will demand more dollars. Accordingly, since demand for dollars can be identified with supply of euros when the only currencies are euros and dollars, Europeans will increase the supply of euros. This shifts the function $S_€$ to the right, thereby causing a fall in the exchange rate: a rise in the European GDP leads to a depreciation of the euro.

Example 10.2. Effect on the equilibrium exchange rate of a rise in the American GDP; see Fig. 12. As shown in Example 10.1, the rise in an economy's GDP causes a depreciation of the domestic currency. Therefore, an increase in the US GDP depreciates the dollar against the euro. Having the dollar depreciated with respect to the euro is equivalent to having the euro appreciated with respect to the dollar. In sum: a rise in the American GDP leads to an appreciation of the euro.

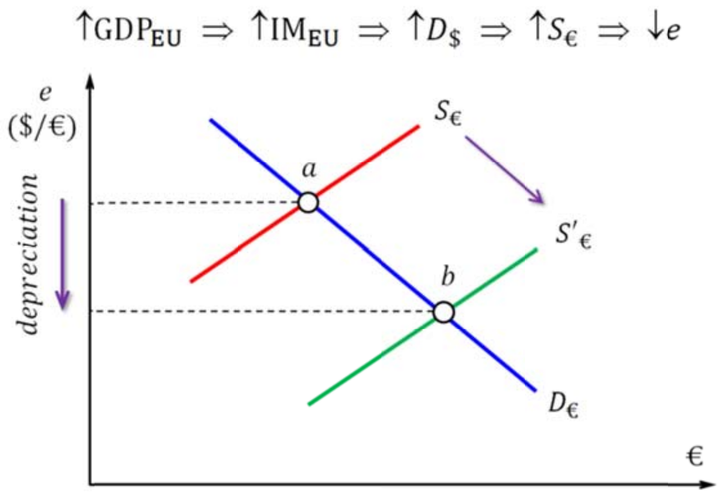

Fig. 11. Effect of a rise in the European GDP

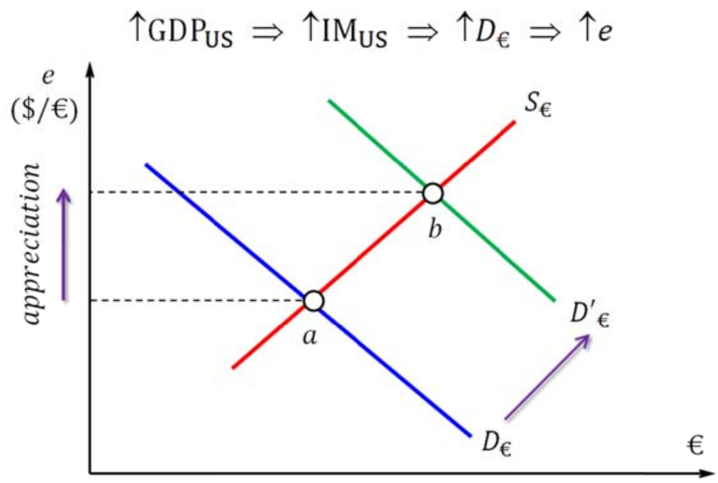

Fig. 12. Effect of a rise in the American GDP

Example 10.3. Effect on the equilibrium exchange rate of a rise in the European inflation rate; see Fig. 13. A rise in the European inflation rate (i) makes American goods comparatively cheaper than American goods to European consumers and (ii) makes European goods comparatively more expensive than American goods to American consumers. Feature (i) encourages European consumers to import more goods from the US, whereas (ii) causes a fall in the US imports from Europe. The increase in European imports from the US shifts the supply of euros function $S_€$ to the right (as Europeans ask for more dollars to purchase more American goods). The reduction in US imports from Europe shifts the demand for euros function $D_€$ to the left (as Americans ask for fewer euros to purchase less European goods). Both shifts lead to a fall in the exchange rate. In sum, a higher European inflation rate depreciates the euro.

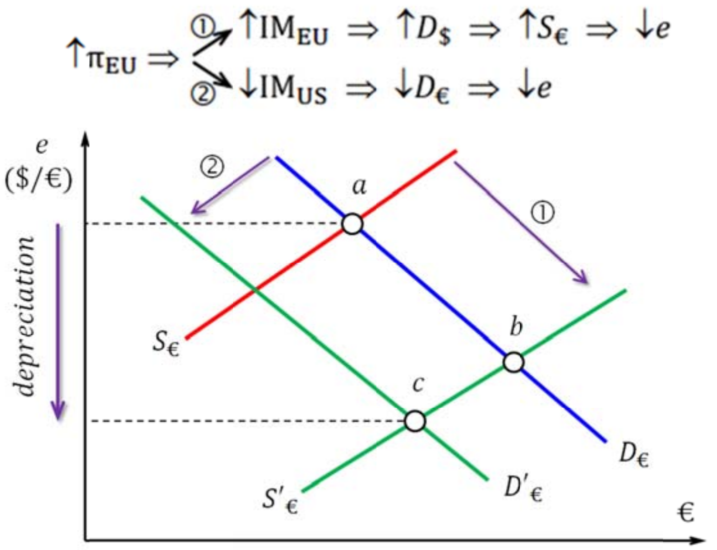

Fig. 13. Effect of a rise in the European inflation rate

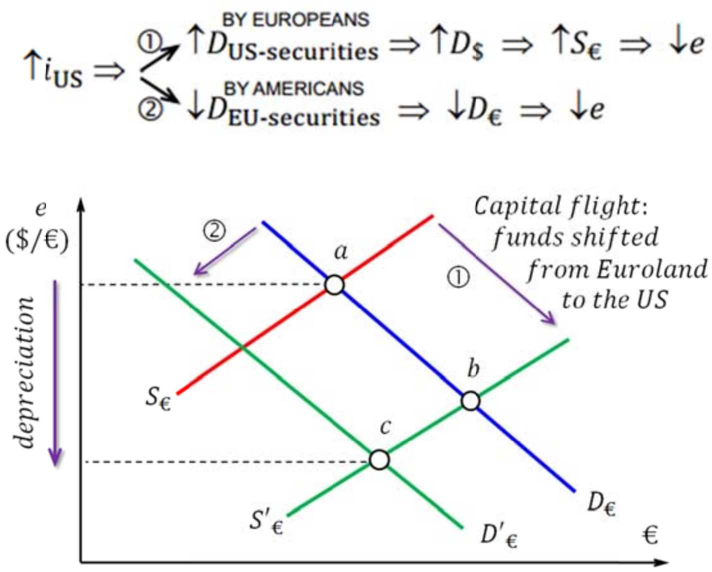

Fig. 14. Effect of a rise in the American interest rate

Example 10.4. Effect on the equilibrium exchange rate of a rise in the American interest rate; see Fig. 14. A rise in the US interest rate (i) makes American financial assets more attractive than European financial assets to European inverstors and (ii) makes European financial assets less attractive than American financial assets to American inverstors. By (i), Europeans increase the demand for American financial assets, the demand for dollars, and the supply of euros, thereby shifting function $S_€$ to the right. By (ii), Americans curtail the demand for European financial assets and reduce accordingly the demand for euros, so function $D_€$ shifts to the left. All in all, a higher US interest rate depreciates the euro.

11. Arbitrage and speculation

Definition 11.1. Arbitrage refers to market transactions that, taking advantage of price differences, generate a sure profit.

Definition 11.2. Speculation is the same as arbitrage with the only difference that transactions do not guarantee a sure profit.

Whereas a speculator is taking a risk, an arbitrageur obtains a risk-free profit. Almost nothing lies outside the scope of arbitration and speculation: commodities, bonds, currencies, shares, options, real estate, derivatives, futures contracts…

12. Spatial arbitrage

Definition 12.1. Spatial arbitrage exploits price differences in different locations.

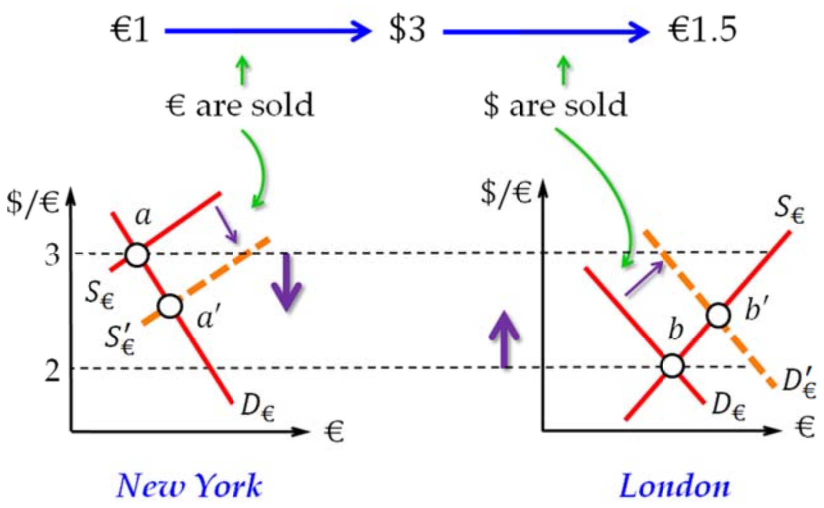

Fig. 15. An example of spatial arbitrage

Example 12.2. Suppose $e_L = 2$ \$/€ in London and $e_N = 3$ \$/€ in New York; see Fig. 15. An arbitrageur would buy euros where they are "cheap" (in London, where buying €1 just takes \$2) to sell them were they are "expensive" (in NY, where you need \$3 to get €1). The sequence

€1 $\rightarrow^{\text{sold in NY}}$ \$3 $\rightarrow^{\text{sold in L}}$ €1.5

generates a sure profit of €0.5 per euro (a 50% profit rate).

The cycle may be continued: €1 → \$3 → €1.5 → \$4.5 → €2.25 → \$6.75 → €3.375 → … These transactions eventually alter prices.

• By buying euros in London, $D_€$ shifts to the right and ↑e in London: the euro appreciates where it is "cheap" (right-hand side of Fig. 15).

• By selling euros in New York, arbitrageurs shift $S_€$ to the right in New York, so ↓e in New York : the euro depreciates where it is "expensive" (left-hand side of Fig. 15).

Summing up, $e_L = 2$ \$/€ rises and $e_N = 3$ \$/€. Eventually (may in a matter of minutes), both prices will converge to some value lying between $e = 2$ and $e = 3$. Reached that value, spatial arbitrage will no longer be possible and, as a result, the exchange rate in both markets, in New York and in London, will be the same.

13. Triangular (or triangle) arbitrage

Definition 13.1. Triangular arbitrage takes advantage of price imbalances involving at least three currencies.

Example 13.2. Let exchange rates be 2 $/€, 3 ¥/$, and 4 ¥/€. Triangular arbitrage can only occur if the product of two rates is not equal to the third one (in the product one of the currencies should cancel out). The second and third cannot be meaningfully multiplied, as no currency cancels out in 3 ¥/$ · 4 ¥/€. By taking the inverse $\frac{1}{3}$ $/¥ of 3 ¥/$ a meaningful product obtains: $\frac{1}{3}$ $/¥ · 4 ¥/€ = $\frac{4}{3}$ $/€ ≠ 2 $/€. This means that there are arbitrage opportunities. There are six exchange sequences:

(1) € → $ → ¥	(2) € → ¥ → $	(3) $ → € → ¥
(4) $ → ¥ → €	(5) ¥ → $ → €	(6) ¥ → € → $.

But (1) is equivalent to both (3) and (5) because all generate the same cycle € → $ → ¥ → €. And (2), (4), and (6) are equivalent because all generate the same cycle € → ¥ → $ → €. As a a consequence, there are two ways of trying to exploit price differences, represented by the two exchange cycles shown below.

One the cycles generates profits; the other, losses. The right-hand cycle yields a loss: €1 → ¥4 → $4/3 → €2/3. The left-hand one produces a profit: €1 → $2 → ¥6 → €1.5.

As noticed, $\frac{\$}{¥} \cdot \frac{¥}{€} < \frac{\$}{€}$: going directly from $ to € is more profitable than going indirectly through ¥. The step "€1 → $2" makes the dollar appreciate, so $/€ falls. The step "$2 → ¥6" makes the yen appreciate, so $/¥ raises. And the step "¥6 → €1.5" makes the euro appreciate, so ¥/€ rises. Hence, the gap between going directly or indirectly tends to close.

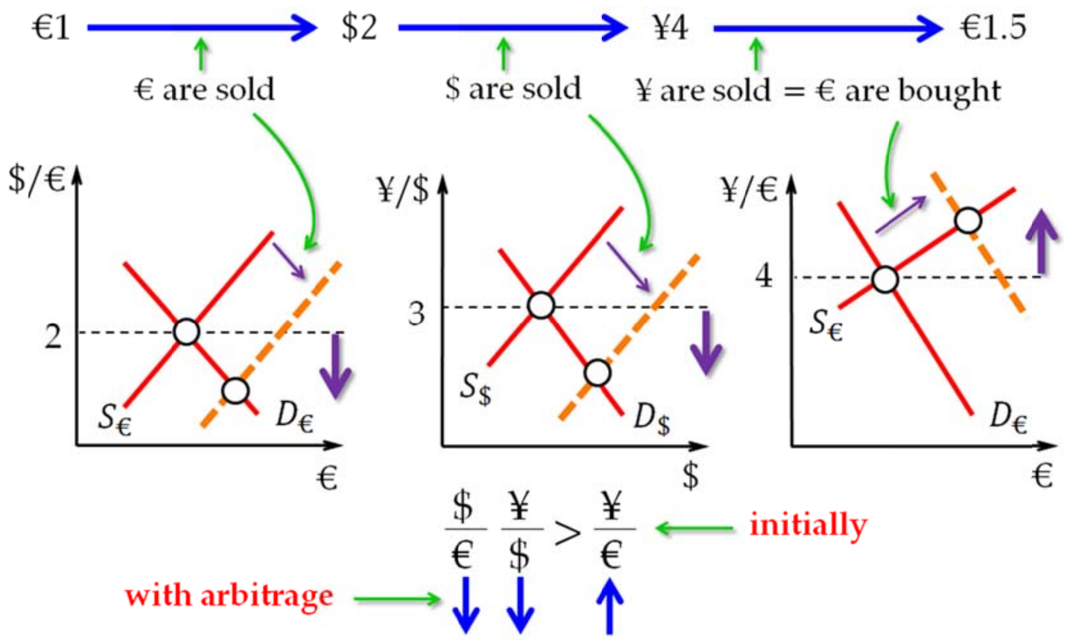

Fig. 16. An example of triangular arbitrage

14. How to become a millionaire in one day

Example 14.1. Let $e = 2$ \$/€ today and suppose I expect $e' = 1.9$ \$/€ tomorrow. Imagine that the overnight (daily) interest rate is 3‰. If my expectation is correct, I can become a millionaire tomorrow.

This is the recipe. I ask for a loan of, say, €100 million. Tomorrow I will have to return this amount plus €300,000. With my €100 million, and given the exchange rate $e = 2$ \$/€, I purchase \$200 million. I could lend those dollars for a day, but since the day has been hard enough I just wait for tomorrow.

Tomorrow comes and I am right. I then sell the \$200 million at the rate $e' = 1.9$ \$/€ and get €105,263,157 (the additional cents, left as a tip). I next repay my €100 million debt plus the loan interest of €300,000. And I finally search for a fiscal paradise that would welcome my remaining €4,963,157…

But what if I am wrong and, for instance, $e' = 2.1$ \$/€? Then I have a little problem, since, at the rate e', I can only obtain €95,238,095.23 from my \$200 million. That means that I incur a loss.

15. Going short vs going long

Definition 15.1. (Wikipedia) "Short-selling […] is the practice of selling assets, usually securities, that have been borrowed from a third party […] with the intention of buying identical assets back at a later date to return to the lender" and make a profit.

"The short seller hopes to profit from a decline in the price of the assets between the sale and the repurchase, as the seller will pay less to buy the assets than the seller received on selling them. Conversely, the short seller will incur a loss if the price of the assets rises."

Definition 15.2. Going long is the strategy opposite to short-selling: an asset is bought expecting that its price will raise.

Example 14.1 illustrates short selling: I assumed a debt in euros because I expected a depreciation of the euro. Hence, by purchasing dollars, I expected to obtain next more euros for the same dollars, so that the debt could be repaid with cheaper euros.

Remark 15.3. To limit market volatility, some restrictions to short selling were imposed in September 2008. In fact, short selling is capable of triggering currency crises.

16. Fixed vs floating exchange rates

There are two basic exchange rate regimes: fixed exchange and floating exchange regime.

Definition 16.1. In a fixed exchange rate regime, the government picks an official value of the exchange rate between the domestic currency and some foreign currency (or group of them) and assumes the compromise of defending that value in the foreign exchange market by buying or selling the domestic currency. If the value of the domestic currency is pegged to the value of another currency, the latter is known as the anchor currency.

Definition 16.2. In a floating (or flexible) exchange rate regime, the government lets the currency market determine the value of the exchange rate.

The remaining regimes combine the previous two in different degrees. Such intermediate regimes are called managed float regime.

Definition 16.3. In the managed float exchange rate regime (or "dirty float") the government seeks to influence the exchange rate by buying and selling currencies at will.

17. Currency market intervention

Let e' be the fixed exchange rate, with the central bank instructed by the government to sustain that value. Imagine that the exchange rate in the market is actually $e < e'$; see point a in Fig. 17. Having e' as fixed exchange rate means that the central bank must intervene to place the market equilibrium along the horizontal line with value e'. The problem is that, at point a, the market does not value the euro as the government intends. The solution is to demand more euros to rise its value.

It may appear that the central bank may either shift $S_€$ to reach point b or shift $D_€$ to reach point c. The first option is not available, since the central bank cannot force a reduction in the supply of euros. What the central bank can do is to expand the demand for euros.

Therefore, to reach value e' from point a, the central bank must demand enough euros to shift the market demand function from $D_€$ to $D'_€$. But the purchase of euros to rise the value from e to e' must be paid in dollars. Accordingly, in passing from a to c, the central bank spends dollars. Obviously, to sell dollars the central bank must have them (or arrange a dollar loan, in general granted by other central banks).

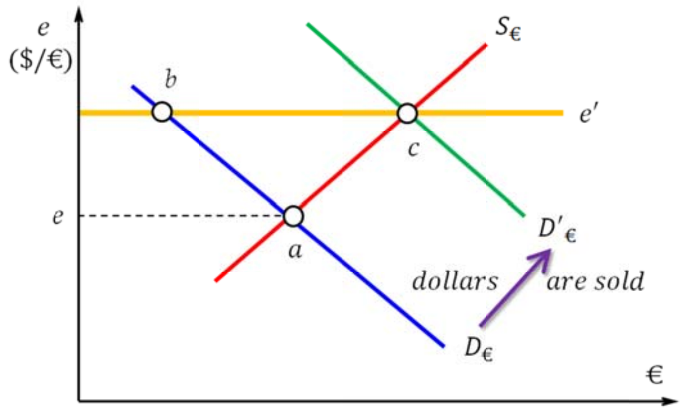

Fig. 17. CB intervention when the currency is undervalued

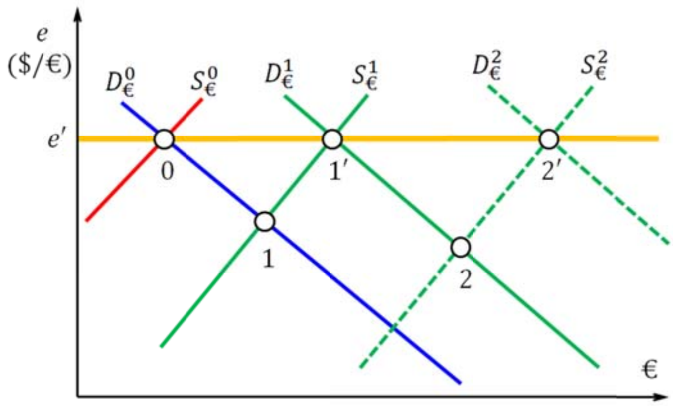

Fig. 18. Trying to avoid a currency crisis

18. Currency crises and speculative attacks

Definition 18.1 (imprecise). A currency crisis typically arises when a fixed exchange rate cannot be defended, that is, achieved through central bank intervention in the currency market.

What if market participants come to believe that a given exchange rate cannot be defended? They will probably engage in short-selling: expecting the euro to lose value, they will ask for loans in euros, and buy dollars with them. That shifts $S_€$ to the right, so the euro loses value indeed. And here it is a self-fulfilling prophecy: what agents do in response to what they expect to occur contributes to cause what they expect to occur.

Example 18.2. Fig. 18 represents the events that hasten a currency crises. The fixed exchange rate is e' and the market is initially at point 0. A speculative attack unfolds through a massive sale of euros, to repurchase them later at a smaller rate. This attack shifts $S_€$ from $S_€^0$ to $S_€^1$, moving the market equilibrium from point 0 to point 1. To defend the fixed rate, the central bank reacts by selling dollars, shifting $D_€$ from $D_€^0$ to $D_€^1$. Market equilibrium then moves from 1 to $1'$.

Excursus 18.3. A priori, a currency is equally likely to appreciate or depreciate. In this respect, mounting a speculative attack without further information is a 50-50 bet, which does not look promising for a speculator. Moreover, in this case, some speculators may bet that the currency is going to depreciate and others that it is going to appreciate, so the two attacks may cancel each other or, in any event, may make defense of the fixed rate easier. Therefore, to ignite a speculative attack some reason must point to a specific unidirectional and persistent modification in the exchange rate. Hence, there must be some objective feature of the economy creating a tendency for the currency to either appreciate or depreciate. That feature will coordinate all speculators to bet in only one direction: either all believe that an appreciation will occur or all believe that it will be a depreciation. Accordingly, a speculative attack is most likely to be conducted for some objective reason whose effects on the currency the attack exacerbates. It is worth noticing as well that it is easier for the central bank to fight a speculative attack based on the bet that the currency is going to appreciate: in that case, to sustain the fixed exchange rate, the CB needs only to sell what it owns in abundance, namely, the domestic currency. Consequently, the archetypal currency crisis will arise when a speculative attack is launched against the currency because some feature in the economy automatically leads the currency to depreciate (for instance, a domestic inflation rate higher than the inflation rate in the rest of the world).

Example 18.2 (continued). A second attack shifts $S_€$ from $S_€^1$ to $S_€^2$, reaching point 2. If the central bank still has enough dollar reserves, equilibrium may be moved to $2'$. If not, the attack is successful and market equilibrium remains at 2: the attack has been successful and has led to a sharp decline in the exchange rate. In this case, the government accepts the new exchange and devaluates the currency (reduces the fixed exchange rate).

19. Revaluation and devaluation

Definition 19.1. A devaluation is a reduction of the fixed exchange rate and occurs when the government accepts that the former fixed rate cannot be upheld as it makes the domestic currency to be overvalued with respect to its market (or sustainable or "fundamental") value.

In Example 18.2, if market participants believe the "right" value to be the one associated with point 2 and the central bank has not enough dollars to sustain any other higher value, declaring the market value to be the new fixed exchange rate means devaluating the exchange rate.

Definition 19.2. A revaluation is the opposite of a devaluation: it is the reset of a fixed exchange rate at a higher value.

Example 19.3. A famous, successful speculative attack. Took place on the 16th of September, 1992: the Black Wednesday. On that date, George Soros became famous for forcing the British government to withdraw from the European Exchange Rate Mechanism (a fixed exchange rate agreement, predecessor of the euro). Soros made a gain of over $1 billion by short selling pound sterlings. Newspapers revealed that the British Treasury spent £27 billion trying to sustain the value of the pound.

20. The impossible trinity (or open economy trilemma)

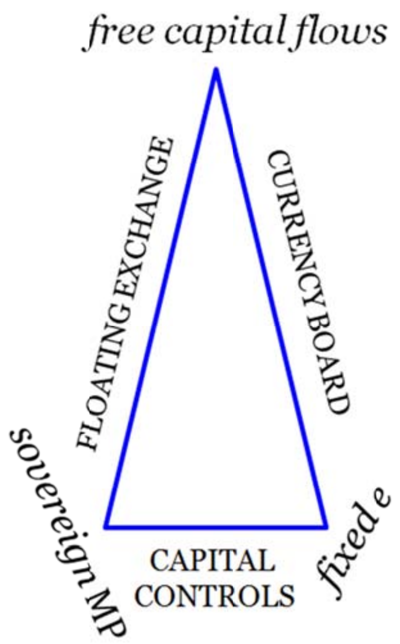

Fig. 19. The impossible trinity

Definition 16.1. Due to Nobel Prize in Economics recipient Robert Mundell, the impossible trinity is the trilemma according to which it is not possible to simultaneously have

- a fixed exchange rate,
- an independent monetary policy, and
- free international capital mobility (that is, no capital control).

The justification of the impossibility is as follows: if *e* is fixed and a monetary policy that expands **M1** is applied, then domestic *i* falls, so *e* falls. To defend *e*, domestic currency must be bought in the currency market, so **M1** is reduced. Fig. 19 on the left depicts the options.

• **Option 1. Floating exchange.** If an independent monetary policy and no capital control are chosen, the exchange rate must float (UK, Canada).

• **Option 2. Currency board.** Opting for fixing the exchange rate and allowing the free mobility of capital implies that monetary policy can no longer be independent. It can be interpreted that the countries in the eurozone have chosen this option: their monetary policy was been handed to a supranational authority, the European Central Bank. When it is a single country that takes this option, the resulting monetary authority is called a currency board, and its goal is merely to adopt the monetary policy of the country (or countries) to which the exchange rate is pegged and be willing to convert into the pegged currency any request of conversion of any amount of domestic currency. Argentina had a currency board in the 1990s when the fixed exchange rate was set as one Argentinian peso per US dollar.

• **Option 3. Capital controls.** If it is chosen to control both the foreign value of the domestic currency, by fixing the exchange rate, and its domestic value (the interest rate), by deciding which monetary policy to conduct, then capital controls must be established (China until recently).

21. Rodrik's trilemma

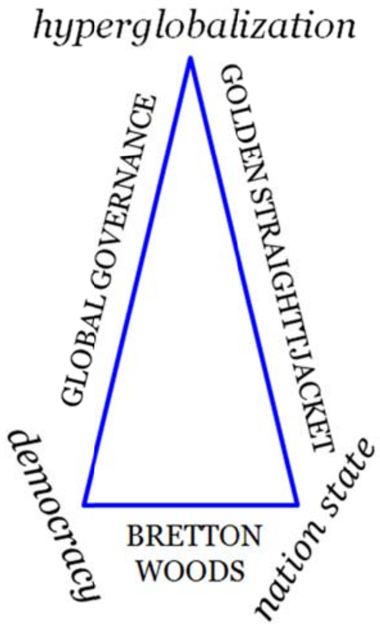

Fig. 20. Rodrik's trilemma

Definition 21.1. The term "globalization" refers to the process and consequences of the opening up of domestic markets (from both the real and the financial sector) to the international markets.

Definition 21.2. Rodrik's fundamental political trilemma of the world economy or the tension between national democracy and global markets; see Fig. 20. "We cannot have hyperglobalization, democracy, and national self-determination all at once […] If we want hyperglobalization and democracy, we need to give up on the nation state. If we must keep the nation state and want hyperglobalization too, then we must forget about democracy. And if we want to combine democracy with the nation state, then it is bye-bye deep globalization." Dani Rodrik (2011): *The globalization paradox: democracy and the future of the world economy*.

There are three options to handle the tension between national democracy and global markets that the trilemma expresses. "We can restrict democracy in the interest of minimizing international transaction costs, disregarding the economic and social whiplash that the global economy occasionally produces. We can limit globalization, in the hope of building democratic legitimacy at home. Or we can globalize democracy, at the cost of national sovereignty." Dani Rodrik, *The globalization paradox*.

• **Option 1. The Golden Straightjacket.** Hyperglobalization means that national borders do not interfere at all with the circulation of goods, services, and capital. If a nation state becomes hyperglobalized, then domestic regulations and taxes must be consistent with the requirements of hyperglobalization and, in particular, with ensuring that the domestic economy remains attractive to, and earns the confidence of, international investors and traders. Therefore, domestic policy must be subordinated to comply with the conditions of economic globalization by adopting such policies as:

- a strict monetary policy ("tight money");
- "flexible" labour markets;
- deregulation, privatization, and minimize public intervention ("small government");
- keep taxes (particularly, capital and corporate taxes) low;
- maintain the economy sufficiently open to the rest of the world ("open borders").

When this set of policies is adopted it is said that the nation state wears The Golden Straightjacket on. The government putting on this jacket is free from domestic (economic or social) obligations or constraints. The requirements of the global economy dictate the domestic policy. Signs of wearing the jacket:

- economic policy-making institutions become "independent" from democratic control (central banks, market regulators);
- social insurance is reduced (privatized);
- corporate taxes and the top income taxes lowered; and
- policy goals are subordinated to keep market confidence.

• **Option 2. Bretton Woods compromise** ("thin" version of globalization). This compromise merely implies a reduced international discipline: each nation state enjoys sufficient freedom to pursue domestic goals, like development levels, as long as restrictions on capital flows are implemented. Since nation states can follow their own paths of development, domestic differences can be maintained and enlarged.

• **Option 3. Global governance**. The global governance option involves removing the nation state in order to have democratic policies and hyperglobalization. This option amounts to relocating politics to the global level, in the sense that rule making becomes supranational (the European Union is a regional example). The difficulties with option 3 emerge from the possibility that there is too much diversity among nation states to make global federalism a practical option.

22. A conjecture

Conjecture 22.1. It is harder for an economy to remain in a (non-spontaneous) state than to achieve it.

Rodrik (*One economics, many recipes: globalization, institutions, and economic growth*, 2007, p. 43) considers the particular case of Conjecture 22.1 when the state of the economy is a growing economy: "Sustaining growth is more difficult than igniting it". Globalization arguably defines another non-spontaneous state to which the Conjecture 22.1 applies: more effort is necessary for an economy to remain globalized than to become globalized. Argentina in the mid-1990s become hyperglobalized very quickly, but the cost of maintaining that state turned out to be unsustainable and led to the catastrophic crisis of 2001.

23. Real exchange rate

Definition 23.1. The real exchange rate e_r is the relative price of the basket of goods defined in the CPI in two economies: e_r is the price of the basket in one economy in terms of the basket of the other. Specifically:

$$e_r = e \cdot \frac{P}{P^*}$$

where e is quoted indirectly, P is the domestic CPI, and P^* is the foreign CPI.

Loosely speaking, the real exchange is the nominal exchange rate expressed in terms of good, where the term "goods" is interpreted as the basket of goods in the CPI. More precisely, the real exchange rate expresses the rate at which the domestic basket (domestic goods) can be exchanged for the foreign basket (foreign goods). Equivalently, the real exchange rate e_r is the nominal exchange e adjusted by the price indices of the two economies. Note that e_r is measured in foreign baskets/domestic basket.

Example 23.2. Suppose $e = 4$ \$/€, $P = 100$ €/basket$_{EU}$, and $P^* = 200$ \$/basket$_{US}$. With these values, how many baskets$_{US}$ could be obtained from one basket$_{EU}$? As $P = 100$, basket$_{EU}$ could be sold for €100. At the rate $e = 4$ \$/€, €100 exchange for \$400. With \$400 and $P^* = 200$, 2 baskets$_{US}$ can be purchased. This says that the purchasing power of 1 basket$_{EU}$ is 2 baskets$_{US}$. That is, $e_r = 2$ baskets$_{US}$/basket$_{EU}$. Applying the formula, $e_r = 4 \cdot 100/200 = 2$ baskets$_{US}$/basket$_{EU}$ ($4 \cdot 100$ is the cost in dollars of the basket$_{EU}$).

24. Competitiveness of an economy

The real exchange real is a measure of competitiveness: the smaller e_r, the higher the competitiveness of the domestic economy.

Example 24.1. In passing from $e_r = 1$ to $e_r = 2$ domestic competitiveness is eroded: with $e_r = 1$, foreigners could obtain a domestic basket with just one of their baskets; with $e_r = 2$, they must deliver 2 of their baskets to get a domestic basket. Going from $e_r = 1$ to $e_r = 2$ means that it is more expensive for foreigners to purchase our basket, so the domestic economy becomes less competitive.

25. Real appreciation and real depreciation

Definition 25.1. A real appreciation is an increase of e_r (a loss of domestic competitiveness).

A real appreciation of the exchange rate means that the domestic basket can buy more foreign baskets: the purchasing power of the domestic basket raises.

Definition 25.2. A real depreciation is a decrease of e_r (an improvement of domestic competitiveness).

A real depreciation of the exchange rate means that the domestic basket can buy fewer foreign baskets: the purchasing power of the domestic basket falls.

26. Purchasing power parity (PPP)

Definition 26.1. Purchasing power parity theory (PPP) is the theory according to which, in the long run, e moves to make e_r equal to 1 and, as a result, one domestic basket exchanges for one foreign basket (both baskets have the same purchase power).

Definition 26.2. The purchasing power parity exchange rate e_{PPP} is the nominal exchange rate e that makes $e_r = 1$; that is, $e_{\mathrm{PPP}} = P^*/P$.

With domestic and foreign baskets being the same, PPP holds that the price of the basket should be the same in both economies when expressed in the same currency: $e \cdot p = P^*$, which holds if $e = e_{\mathrm{PPP}}$.

Definition 26.3. If $e > e_{\mathrm{PPP}}$, then domestic currency is said to be overvalued (with respect to its parity value). If $e < e_{\mathrm{PPP}}$, it is said to be undervalued. The percentage of overvaluation is $\frac{e - e_{\mathrm{PPP}}}{e_{\mathrm{PPP}}}$.

Example 26.4. With $p^* = \$100$, $p = €50$, and $e = 4$ \$/€, the euro is overvalued with respect to the dollar. In fact, $e_{\mathrm{PPP}} = p^*/p = 100/50 = 2$ \$/€. This is reasonable: since the price of a book in the US doubles the price of a book in Euroland, purchasing power parity demands that €1 be capable of purchasing \$2.

Having $e = 4$ instead of $e = 2$ implies that the euro has more purchasing power than it should : with €50, one book can be bought in Euroland; given $e = 4$, €50 can buy 2 books in the US. The euro is the a 100% overvalued: $\frac{e - e_{\mathrm{PPP}}}{e_{\mathrm{PPP}}} = \frac{4-2}{2} = 1 = 100\%$.

27. PPP and commercial arbitrage

Definition 27.1. Commercial arbitrage consists of buying goods where they are cheap and selling them where they are expensive.

In the absence of transportation costs, PPP can be justified by commercial arbitrage.

Example 27.2. Suppose that only one good can be traded between Euroland and the US: Macroeconomic textbooks. The price of a textbook in the US is $p^* = \$100$; in Euroland, $p = €50$. Letting $e = 4$ \$/€, the price in dollars of a Euroland textbook is 4 \$/€ · €50 = \$200.

Consequently, textbooks are cheap in the US. Commercial arbitrageurs would proceed as shown in Fig. 21 (it is assumed that textbooks can be sent from one economy to the other at no transport cost).

- If the arbitrageur is an American, then he or she will buy textbooks in the US to subsequently ship them to Euroland; once sold there, euros are converted into dollars.

- If the arbitrageur is a European, then he or she will first convert euros are converted into dollars, buy textbooks in the US to finally ship them to Euroland and sell them there.

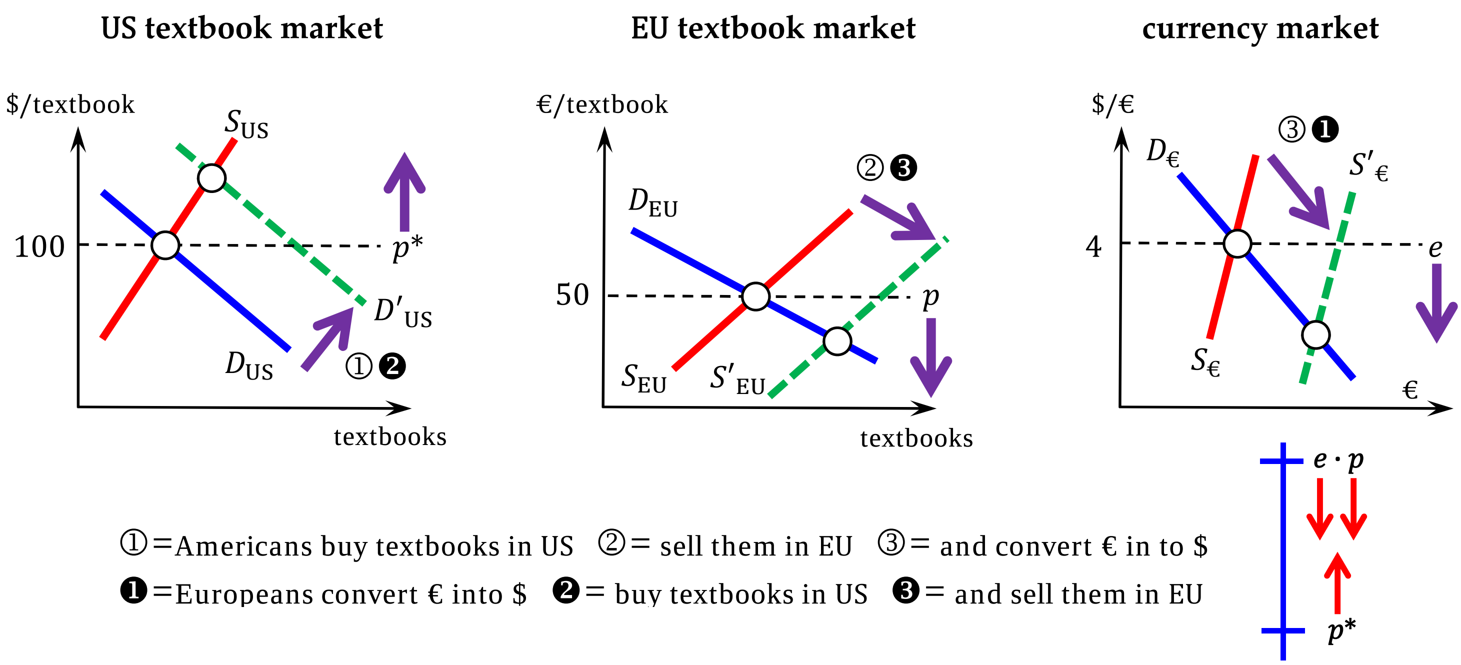

Fig. 21. Commerical arbitrage and purchasing power parity

The purchase of books in the US tends to <u>rise</u> p^*. The sale of those books in Euroland make p <u>fall</u>. The rise in the demand for dollars induces a <u>reduction in e</u>. Initially, $4{\cdot}50 = e \cdot p > p^* = 100$. Thanks to arbitrage, $e \cdot p$ tends to fall and p^* tends to rise. Eventually, $e \cdot p = p^*$. This condition stops arbitrage and makes e reach the PPP value p^*/p .

28. The Big Mac index

Definition 28.1. The Big Mac (BM) index is an index defined by *The Economist* to test the purchasing power parity theory by choosing the Big Mac as the basket of goods.

Big MacCurrencies

Hamburger prices round the world

Country	Price* in local currency	Implied† purchasing power parity of the dollar	Actual exchange rate Sept 1st	% over (+) or under (−) valuation of US$
Australia	A\$1.75	1.09	1.64	+50
Belgium	BFr90	56	42	−25
Brazil	Cz\$2.5	7.80	13.80	+78
Britain	£1.10	0.69	0.67	−3
Canada	C\$1.89	1.18	1.39	+18
France	FFr16.4	10.30	6.65	−35
Hongkong	HK\$7.60	4.75	7.80	+64
Ireland	IR£1.18	0.74	0.74	−1
Japan	Y370	231	154	−33
Holland	Fl4.35	2.72	2.28	−16
Singapore	S\$2.80	1.75	2.15	+23
Spain	Ptas260	163	133	−18
Sweden	SKr16.5	10.30	6.87	−33
United States	\$1.60	—	—	—
W Germany	DM4.25	2.66	2.02	−24

*Source: McDonald's. *Prices may vary slightly between branches. †Foreign price divided by dollar price.*

Fig. 22. BM index, 6 Sept 1986

http://www.economist.com/news/business-and-finance/21639762-our-article-1986-introducing-big-mac-index-origins-hamburger-standard

Example 28.2. The first BM index chart: 6 September 1986. The first BM index chart is shown in Fig. 22. Take Spain as the home economy. The price in Spain of a BM was $p = 260$ Ptas/BM. The price in the US of a BM was $p^* = 1.60$ USD/BM. Therefore,

$$e_{\text{PPP}} = \frac{p*}{p} = \frac{1.60\ \text{USD/BM}}{260\ \text{Pts/BM}} = \frac{2\ \ \text{USD}}{325\ \text{Ptas}} = \frac{325\ \ \text{Pts}}{2\ \ \text{USD}} = 162.5\frac{\text{Pts}}{\text{USD}}$$

The market exchange rate was $e = 133\frac{\text{Pts}}{\text{USD}} < e_{\text{PPP}} = 162.5\frac{\text{Pts}}{\text{USD}}$. As a result, the peseta was undervalued. The exact amount is

$$\%\ overvaluation = \frac{e - e_{\text{PPP}}}{e_{\text{PPP}}} \cdot 100 = \frac{133 - 162.5}{162.5} \cdot 100 = -18.15\%.$$

According to PPP, one would have expected the peseta to eventually appreciate against the dollar.

Fig. 23. BM index, January 2014

Economist.com

Figs. 23 and 24 provide the last three estimates (Jan 2014, Jul 2014, Jan 2015) of the degree of overvaluation or undervaluation with respect to the dollar of several currencies according to the Big Mac index.

http://www.economist.com/news/finance-and-economics/21595037-our-bun-loving-guide-currencies-grease-proof-taper

http://www.economist.com/news/finance-and-economics/21640370-some-currencies-lose-weight-diet-qe-and-cheap-oil-others-bulk-up-oily-and

Fig. 24. BM index, January 2015

US = home economy	P^* = BM home price	BM price in \$ = P^*/e	PPP of \$ = P^*/P	market e on 22 Jan 2014	Over (+) under (–) valuation against the \$
US	P = 4.62	–	–	–	–
Brazil	12.4 R\$	5.25	2.68	2.36	13.48
UK	2.79 £	4.63	1.66 \$/£	1.66 \$/£	0.06
China	16.6 ¥	2.74 \$	3.59 ¥/\$	6.05 ¥/\$	-40.68
Eurozone	3.65 €	4.96	1.26 \$/€	1.35 \$/€	7.3
India	95 ₹	1.54	20.54	61.85	-66.78
Russia	89 руб	2.62	19.25	33.94	-43.29
Sweden	40.7 kr	6.29	8.8	6.47	35.97
Venezuela	45 Bs.F.	7.15	9.73	6.29	54.66

Fig. 25. BM index, 22 Jan 2014 | http://bigmacindex.org/

Fig. 25 provides detailed information to determine the degree of under or overvaluation of several currencies with respect to the dollar according to the BM index on the 22nd of January, 2014.

Example 28.3. In Venezuela, the BM was priced at Bs. F. 45. Given the market rate of 6.29 Bs. F./\$, the price in dollars of a BM is \$7.15. If PPP between Bs. F. and dollar held, it should had been \$4.62 (the US price). With respect to the dollar, the Bs. F. is overvalued a $\frac{7.15-4.62}{4.62} \approx 54\%$.

Example 28.4. Given the market exchange rate in January 2014, the dollar price of the BM in China was \$2.74. In the US, the price was \$4.62. The yuan was undervalued a $\frac{2.74-4.62}{4.62} \approx 40\%$ against the dollar.

PPP predicted that the Venezuelan bolivar would eventually depreciate and the yuan appreciate. In the China case, undervaluation results from the comparison of P (the US price of the BM) with P^*/e (the China price of the BM given the exchange rate). Undervaluation follows from the fact that $P^*/e < P$, since $2.74 < 4.62$. Adopting the Chinese perspective as domestic, $1/e < P/P^*$. Whereas $1/e$ is the market exchange rate (in \$/¥ units), P/P^* is the PPP exchange rate (in \$/¥ units as well). In fact, $1/e$ and P/P^* can be used to calculate undervaluation:

$$\%undervaluation = 100 \cdot \frac{1/e \ - \ P/P^*}{P/P^*} \approx 100 \cdot \frac{1/6.05 \ - \ 4.62/16.6}{4.62/16.6} \approx 100 \cdot \frac{0.165 - 0.278}{0.278} \approx 40.6\%.$$

US = home economy	P^* = BM home price	BM price in \$ = P^*/e	PPP of \$ = P^*/P	market e on 22 Jan 2015	Over (+) under (–) valuation against the \$
US	P = 4.79	–	–	–	–
Brazil	13.5 R\$	5.21	2.82	2.59	8.70
UK	2.79 £	4.63	1.66 \$/£	1.66 \$/£	0.06
China	17.2 ¥	2.77 \$	3.59 ¥/\$	6.21 ¥/\$	–42.19
Eurozone	3.68 €	4.26	0.77 \$/€	0.86 \$/€	–10.98
India	116.25 ₹	1.89	24.27	61.62	–60.61
Russia	89 руб	1.36	18.58	65.23	–71.51
Sweden	40.7 kr	4.97	8.50	8.19	3.73
Switzerland	6.5 CHF	7.54	1.36	0.86	57.49
Venezuela	132 Bs.F.	2.53	27.56	52.10	–47.119

Fig. 26. BM index, 22 Jan 2015 | http://bigmacindex.org/

Fig. 26 shows the information underlying the BM index in January 2015. It can be used to test the prediction originated in the analysis of the data in Fig. 25. For instance, as predicted, the bolivar has depreciated a 728.29% between January 2014 (6.29 Bs. F./\$) and January 2015 (52.10 Bs. F./\$).

As regards China, not as predicted, the yuan has appreciated: from 6.05 ¥/\$ (January 2014) to 6.21 ¥/\$ (January 2015). Since the yuan-dollar exchange rate is quoted directly, the change represents a 2.64% depreciation.

The data in Fig. 26 indicate that, according to the PPP theory based on the BM index, currencies that should be expected to depreciate are the Brazilian real, the British pound (slightly), the Swedish crown, and, above all, the Swiss franc (almost 58% overvalued). The currencies expect to appreciate include the Chinese yuan, the euro, the Indian rupee, the Venezuelan bolivar, and, most significantly, the Russian rouble (undervalued by more than a 70%). Those failing to pass the course will see next year.

29. Relative purchasing power parity

Definition 29.1. Relative purchasing power parity, the dynamic version of the (absolute) purchasing power parity, holds that the exchange rate moves to neutralize inflation differentials. Concretely, define the rate of appreciation of the exchange rate between two currencies as $\hat{e} = \frac{e - e_{-1}}{e_{-1}}$, where e is the current value of the exchange rate and e_{-1} is its value in the immediately preceding period. Let π denote the domestic inflation rate, and π^* the foreing inflation rate, between these two periods. The exact version of the parity is given by (1), whereas its common formulation, an approximation of (1), is given by (2).

$$1 + \hat{e} = \frac{1 + \pi^*}{1 + \pi} \tag{1}$$

$$\hat{e} \approx \pi^* - \pi \tag{2}$$

If the euro is the domestic currency, the dollar is the foreign currency, and the units of e are \$/€, then (2) asserts that the rate of appreciation of the euro is approximately equal to the difference between the US inflation rate and the European inflation rate.

Example 29.2. Suppose $\pi^* = 5\%$ and $\pi = 25\%$. Then, by (2), $\hat{e} \approx 5 - 25 = -20\%$: the euro must depreciate by 20% to compensate for the fact that European prices grow 20 points faster than American prices.

Remark 29.3. Absolute PPP implies relative PPP but not vice versa. In fact, if absolute PPP holds, then (1) can be obtained as follows.

$$1 + \hat{e} = 1 + \frac{e - e_{-1}}{e_{-1}} = 1 + \frac{e}{e_{-1}} - 1 = \frac{e}{e_{-1}} \overset{\text{PPP}}{=} \frac{\frac{P^*}{P^*_{-1}}}{\frac{P}{P_{-1}}} = \frac{\frac{P^*}{P}}{\frac{P^*_{-1}}{P_{-1}}} = \frac{1 + \frac{P^*}{P} - 1}{1 + \frac{P^*_{-1}}{P_{-1}} - 1} = \frac{1 + \pi^*}{1 + \pi}$$

30. The uncovered interest rate parity

Definition 30.1. The uncovered interest rate parity establishes a relationship between the domestic interest rate i, the foreign interest rate i^*, and the expected rate of appreciation $\hat{e}^e = \frac{e - e}{e}$ of the domestic currency with respect to the foreign currency. The exact version of the uncovered interest rate parity is given by (3), whereas its usual formulation is given by (4) (taxes are expressed in per one terms).

$$\hat{e}^e = \frac{i^* - i}{1 + i} \tag{3}$$

$$\hat{e}^e \approx i^* - i \tag{4}$$

Equation (3) can be justified by the equality of returns from investing domestically and investing abroad. Specifically, suppose a investor has €1 to lend in period t so that the loan is repaid in $t + 1$. The domestic interest rate between t and $t + 1$ is i. The foreign interest rate between t and $t + 1$ is i^*. The exchange rate in t is e \$/€. The investor faces at least two options; see Fig. 27.

- Option 1. To lend the euro at home in t. In this case, in $t + 1$, the investor receives €$(1 + i)$.

- Option 2. To lend the euro abroad in t. This option involves exchanging the euro for e dollars (since the exchange rate is e \$/€) in order to next lend the e dollars abroad. As a result, in $t + 1$, the investor receives \$$(1 + i^*)\cdot e$. If, in t, the investor expects the exchange rate in $t + 1$ to be e^e, then the investor expects to obtain €$(1 + i^*)\cdot e/e^e$.

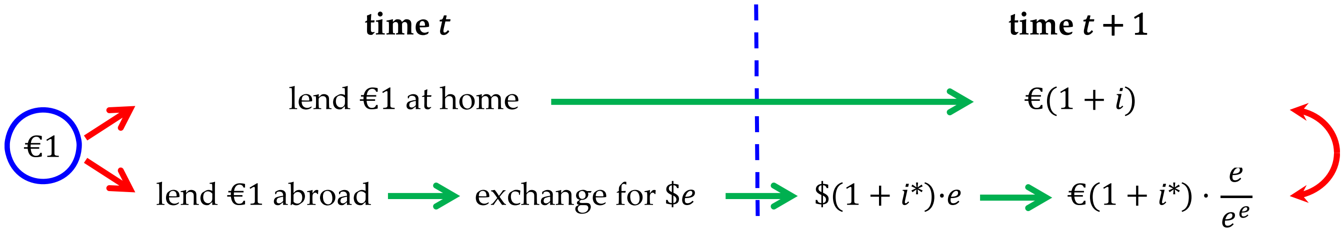

Fig. 27. Justification of the uncovered interest rate parity

The presumption that both options produce the same result implies that

$$1 + i = (1 + i^*)\cdot \frac{e}{e^e}$$

or, equivalently,

$$\frac{e}{e^e}\cdot(1 + i) = 1 + i^*$$

which is equivalent to

$$\left(1 + \frac{e}{e^e} - 1\right)\cdot(1 + i) = 1 + i^*$$

That is,

$$(1 + \hat{e}^e)\cdot(1 + i) = 1 + i^*$$

Therefore,

$$1 + \hat{e}^e + i + \hat{e}^e\cdot i = 1 + i^*$$

Solving for $\hat{e}^e$ leads to (3). The approximation (4) of (3) follows from the assumption that i is small enough. For sufficiently small i, $1 + i \approx i$ and (4) approximates (3).

An interpretation of (4) is as follows. Suppose the foreign interest rate i^* abroad is larger than the domestic interest rate i. This means that $i^* - i > 0$. The interest parity condition (4) contends that, in this case, an appreciation of the domestic currency should be expected: $\hat{e}^e > 0$ (which means that $e^e > e$). This appreciation is required to compensate for the fact that investing abroad is, in terms of interest rates, more profitable. In other words, the higher return obtained by investing abroad is reduced by a loss when converting the foreign currency back into the domestic currency, so that the net result is the same as if investment had been at home.

Remark 30.2. The term "uncovered" refers to the fact that e^e is an expectation, not an actual value: the option of investing abroad is not covered against the risk of predicting the exchange rate wrongly.

If the domestic interest is higher than the foreign interest, $i^* - i < 0$, then what (4) demands is to expect a depreciation of the domestic currency: $\hat{e}^e < 0$ (that is, $e^e < e$).

Example 30.3. Suppose $i = 5\%$ and $i^* = 25\%$. Then, by (4) , it must be that $\hat{e}^e \approx i^* - i = 0.25 - 0.05 = 0.2 = 20\%$: the expectation should be that domestic currency will appreciate by 20%. According to the exact version (3) of the interest parity, $\hat{e}^e = \frac{i^* - i}{1 + i} = \frac{0.25 - 0.05}{1.05} = \frac{0.20}{1.05} = \frac{20}{105} = \frac{4}{51} \approx 0.238 = 23.8\%$. Since 5% is not a small value for the domestic interest, (3) and (4) differ significantly.

Remark 30.4. If parities (2) and (4) hold, and if expectations are correct, $\hat{e} \approx \pi^* - \pi$, $\hat{e}^e \approx i^* - i$, and $\hat{e}^e = \hat{e}$. Hence, $\pi^* - \pi \approx i^* - i$: <u>the inflation differential between countries reflects the interest differential</u>.