

## Overlapping generations with a public good

### The economy

1. Each generation 0 has 100 members: 50 of them (“the poor”) with endowment  $(1, 0)$  [meaning that they have one unit of the good when young and zero units when old] and the other 50 (“the rich”) with endowment  $(4, 1)$ .
2. There is no capital nor production: people spend their endowments in consumption  $c$ , loans  $l$ , and contributions  $e$  to a public good.
3. The public good only benefits the young individuals. Consequently, the old make no contribution to the public good. The utility function of each young individual  $i$  is  $u_t^i = c_t^i(t) \cdot c_t^i(t+1) \cdot [1 + g(\sum_{j \in N(t)} e^j)]$ , where  $e^j$  is the contribution by young individual  $j$ . It is assumed that  $e^j$  cannot be negative nor higher than  $j$ 's endowment when young.
4. For simplicity, suppose  $g(\sum_{j \in N(t)} e^j) = \sum_{j \in N(t)} e^j$ . This can be viewed as the production function of the public good: the total amount of contributions  $\sum_{j \in N(t)} e^j$  generate the amount  $g(\sum_{j \in N(t)} e^j)$  of public good.
5. The intuition is that each unit of public good reinforces the utility of private consumption: the public good makes the consumption of the good more useful.
6. Question: what is the amount of contribution to the public good made, in equilibrium, by a poor individual (call this contribution  $e^P$ ) and what the amount by a rich individual (call it  $e^R$ )?