Overlapping generations with theft

The economy

1. Each generation 0 has 100 members: 80 of them ("the poor") with endowment $(1, 0)$ and the other 20 ("the rich") with endowment $(4, 2)$. Each component of the endowment vectors is amount of the only good in the economy.

2. All (young) members of all generations have utility function $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$.

3. There is no capital nor production.

4. The young poor can, and are willing to steal, good from the rich. Specifically, after stealing from the rich, each young poor gets b units of the good. The total theft amounts to $80 \cdot b$ units, which are obtained as follows: $3 \cdot b$ units are taken from each young rich, whereas b units are taken from each old rich. Hence, what the poor obtain ($80 \cdot b$) equals what the rich lose ($20 \cdot 3 \cdot b + 20 \cdot b$).

5. Compare the individual and group consumption vectors that arise in equilibrium with and without theft (suppose $b = 1$). Does theft increase or decrease inequality?

No theft analysis

Budget constraint of a young poor individual

$$c_t^{i,P}(t) + l^{i,P}(t) = 1$$

Budget constraint of an old poor individual

$$c_t^{i,P}(t+1) = R(t) \cdot l^{i,P}(t)$$

Lifetime budget constraint of a poor individual

$$c_t^{i,P}(t) + \frac{c_t^{i,P}(t+1)}{R(t)} = 1$$

Budget constraint of a young rich individual

$$c_t^{i,R}(t) + l^{i,R}(t) = 4$$

Budget constraint of an old, initially rich, individual

$$c_t^{i,R}(t+1) = 2 + R(t) \cdot l^{i,R}(t)$$

Lifetime budget constraint of a rich individual

$$c_t^{i,R}(t) + \frac{c_t^{i,R}(t+1)}{R(t)} = 4 + \frac{2}{R(t)}$$

Consumption function of a young poor individual

To maximize $u_t^{i,P}$, $MRS_t^{i,P} = R(t)$. Hence, $c_t^{i,P}(t) = \frac{c_t^{i,P}(t+1)}{R(t)}$. Using the lifetime budget constraint, $2 \cdot c_t^{i,P}(t) = 1$. The demand function for consumption of a young poor individual is therefore $c_t^{i,P}(t) = \frac{1}{2}$.

Savings of a poor individual

The savings function of a young poor individual is $s^{i,P}(t) = 1 - c_t^{i,P}(t) = 1 - \frac{1}{2} = \frac{1}{2}$.

Total savings of the poor individuals

Since there are 80 young poor individuals in period t, total savings $S^P(t)$ of the poor are

$$S^P(t) = 80 \cdot s^{i,P}(t) = 80 \cdot \frac{1}{2} = 40\,.$$

Consumption function of a young rich individual

Since $c_t^{i,R}(t) = \frac{c_t^{i,R}(t+1)}{R(t)}$, the demand function for consumption of a young rich individual is

$$c_t^{i,R}(t) = 2 + \frac{1}{R(t)}\,.$$

Savings of a rich individual

The savings function of a young rich individual is

$$s^{i,R}(t) = 4 - c_t^{i,R}(t) = 4 - \left(2 + \frac{1}{R(t)}\right) = 2 - \frac{1}{R(t)}.$$

Total savings of the rich individuals

Since there are 20 young rich individuals in period t, total savings are

$$S^R(t) = 20 \cdot s^{i,R}(t) = 40 - \frac{20}{R(t)}\,.$$

Total savings

The total savings function is $S(t) = S^P(t) + S^R(t)$. Hence,

$$S(t) = 40 + \left(40 - \frac{20}{R(t)}\right) = 80 - \frac{20}{R(t)}$$

Equilibrium condition

$$S(t) = 0$$

Equilibrium interest rate

Solving $80 - \frac{20}{R(t)} = 0$ for $R(t)$ yields $R(t) = 1/4$, which is the equilibrium interest rate.

Loans in equilibrium

The poor lend, in aggregate, $S^P(t) = 40$ in period t (they must save for the old age, a time when they have no endowment). This is the amount that the rich borrow at t: $S^R(t) = 40 - \frac{20}{R(t)} = 40 - \frac{20}{1/4} = 40 - 80 = -40$ (each rich individual borrows 2 units of the good). This means that, through the loan market, the rich get richer at t: the savings of the poor at t make the rich individuals richer at t. The poor lend 40 at t, but receive only 10 at $t + 1$.

Equilibrium consumption vectors: the poor

The consumption vector of each poor individual is $\left(c_t^{i,P}(t), c_t^{i,P}(t+1)\right) = \left(\frac{1}{2}, \frac{1}{8}\right)$. The corresponding utility is $u_t^{i,P} = c_t^{i,P}(t) \cdot c_t^{i,P}(t+1) = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$. The total consumption vector is $\left(c_t^P(t), c_t^P(t+1)\right) = (40, 10)$.

Equilibrium consumption vectors: the rich

The consumption vector of each rich individual is $\left(c_t^{i,R}(t), c_t^{i,R}(t+1)\right) = \left(6, \frac{3}{2}\right)$. This confirms the claim that the rich get richer at t: without the loan market, each rich individual could at t consume at most 4 (his endowment), but now he consumes 6. The corresponding utility is $u_t^{i,R} = c_t^{i,R}(t) \cdot c_t^{i,R}(t+1) = 6 \cdot \frac{3}{2} = 9$. The total consumption vector is $\left(c_t^R(t), c_t^R(t+1)\right) = (120, 30)$.

Theft analysis

Budget constraint of a young poor individual

$$c_t^{i,P}(t) + l^{i,P}(t) = 1 + b$$

Budget constraint of an old poor individual

$$c_t^{i,P}(t+1) = R(t) \cdot l^{i,P}(t)$$

Lifetime budget constraint of a poor individual

$$c_t^{i,P}(t) + \frac{c_t^{i,P}(t+1)}{R(t)} = 1 + b$$

Budget constraint of a young rich individual

$$c_t^{i,R}(t) + l^{i,R}(t) = 4 - 3 \cdot b$$

Budget constraint of an old, initially rich, individual

$$c_t^{i,R}(t+1) = 2 - b + R(t) \cdot l^{i,R}(t)$$

Lifetime budget constraint of a rich individual

$$c_t^{i,R}(t) + \frac{c_t^{i,R}(t+1)}{R(t)} = 4 - 3 \cdot b + \frac{2 - b}{R(t)}$$

Consumption function of a young poor individual

To maximize $u_t^{i,P}$, $MRS_t^{i,P} = R(t)$. Hence, $c_t^{i,P}(t) = \frac{c_t^{i,P}(t+1)}{R(t)}$. Using the lifetime budget constraint, $2 \cdot c_t^{i,P}(t) = 1 + b$. The demand function for consumption of a young poor individual is therefore $c_t^{i,P}(t) = \frac{1+b}{2}$.

Savings of a poor individual

The savings function of a young poor individual is $s^{i,P}(t) = 1 + b - c_t^{i,P}(t) = 1 + b - \frac{1+b}{2} = \frac{1+b}{2}$.

Total savings of the poor individuals

Since there are 80 young poor individuals in period t, total savings $S^P(t)$ of the poor are

$$S^P(t) = 80 \cdot s^{i,P}(t) = 80 \cdot \frac{1+b}{2} = 40 \cdot (1+b)\,.$$

Consumption function of a young rich individual

Since $c_t^{i,R}(t) = \frac{c_t^{i,R}(t+1)}{R(t)}$, the demand function for consumption of a young rich individual is

$$c_t^{i,R}(t) = 2 - \frac{3}{2} \cdot b + \frac{2-b}{2 \cdot R(t)}\,.$$

Savings of a rich individual

The savings function of a young rich individual is

$$s^{i,R}(t) = 4 - c_t^{i,R}(t) = 4 - 3 \cdot b - \left(2 - \frac{3}{2} \cdot b + \frac{2-b}{2 \cdot R(t)}\right) = 2 - \frac{3 \cdot b}{2} - \frac{2-b}{2 \cdot R(t)}.$$

Total savings of the rich individuals

Since there are 20 young rich individuals in period t, total savings are

$$S^R(t) = 20 \cdot s^{i,R}(t) = 40 - 30 \cdot b - \frac{20 - 10 \cdot b}{R(t)}\,.$$

Total savings

The total savings function is $S(t) = S^P(t) + S^R(t)$. Hence,

$$S(t) = [40 \cdot (1+b)] + \left(40 - 30 \cdot b - \frac{20 - 10 \cdot b}{R(t)}\right) = 80 + 10 \cdot b - \frac{20 - 10 \cdot b}{R(t)}$$

Equilibrium condition

$$S(t) = 0$$

Equilibrium interest rate

Solving $880 + 10 \cdot b - \frac{20-10 \cdot b}{R(t)} = 0$ for $R(t)$ yields the equilibrium interest rate $R(t) = \frac{20-10 \cdot b}{80+10 \cdot b}$.

Loans in equilibrium when $b = 1$

The poor lend, in aggregate, $S^P(t) = 40 \cdot (1 + b) = 80$ in period t. The equilibrium interest rate is $R(t) = \frac{20-10\cdot b}{80+10\cdot b} = \frac{1}{9}$ (with respect to the no theft case, R falls) This is the amount that the rich borrow at t: $S^R(t) = 40 - 30 \cdot b - \frac{20-10\cdot b}{R(t)} = 10 - 90 = -80$ (each rich individual borrows 4 units of the good). Despite theft, the rich get even richer at t: though the poor steal $20 \cdot 3 \cdot b + 20 \cdot b = 80$, they lend also 80 to receive $\frac{1}{9} \cdot 80 \approx 8.88$ in the next period.

Equilibrium consumption vectors: the poor

The consumption vector of each poor individual is $\left(c_t^{i,P}(t), c_t^{i,P}(t+1)\right) = \left(1, \frac{1}{9}\right)$. The corresponding utility is $u_t^{i,P} = c_t^{i,P}(t) \cdot c_t^{i,P}(t+1) = 1 \cdot \frac{1}{9} = \frac{1}{9}$. The total consumption vector is $\left(c_t^P(t), c_t^P(t+1)\right) = \left(80, \frac{80}{9}\right)$. It is worth noticing that the poor's consumption under theft when old $(\frac{1}{9})$ is smaller than their consumption without theft $(\frac{1}{8})$. Paradoxically, stealing from the rich when young makes the poor worse off when old.

Equilibrium consumption vectors: the rich

The consumption vector of each rich individual is $\left(c_t^{i,R}(t), c_t^{i,R}(t+1)\right) = \left(5, \frac{5}{9}\right)$. The corresponding utility is $u_t^{i,R} = c_t^{i,R}(t) \cdot c_t^{i,R}(t+1) = 5 \cdot \frac{5}{9} = \frac{25}{9}$. The total consumption vector is $\left(c_t^R(t), c_t^R(t+1)\right) = \left(100, \frac{100}{9}\right)$. Theft lowers the rich's welfare when young as well as when old. As a result, theft only benefits the poor when young: the poor when old, the rich when young, and the rich when old are all worse off when theft occurs.

Summary of results: total consumption and individual utilities when $b = 1$

total consumption	t			$t+1$		
	no market	market & no theft	market & theft	no market	market & no theft	market & theft
poor	80 $u^i = 0$	40 $u^i = 1/16$	80 $u^i = 1/9$	0	10	$80/9 \approx 8.88$
rich	80 $u^i = 8$	120 $u^i = 9$	100 $u^i = 25/9$	40	30	$100/9 \approx 11.1$

Suggestion: extend the resuts of the previous table when $b = 1/2$ and when $b = 2$.