

# Autarky more likely than complete specialization

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## Abstract

This paper measures the strength of comparative advantage as a source of international trade by forcing countries to choose between two extreme situations: autarky and complete specialization. It is shown in the standard Ricardian model with two countries, two commodities, one input, and linear production possibilities frontiers that autarky is more likely than complete specialization when: (i) for each country, no point on a production possibilities frontier is more likely to be chosen in autarky than another one; and (ii) trade occurs if and only if both countries can increase the consumption of each commodity.

*Keywords:* Autarky, comparative advantage, linear production possibilities frontier, Pareto principle, probability of trading.

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## 1. Introduction

Ricardo's (1821, ch. 7) analysis of foreign trade seems to suggest that countries should specialize completely: "Under a system of perfectly free commerce, each country naturally devotes its capital and labour to such employments as are most beneficial to each. (...) It is this principle which determines that wine shall be made in France and Portugal, that corn shall be grown in America and Poland, and that hardware and other goods shall be manufactured in England."

Textbooks on the principles of economics tend to be somewhat ambiguous when explaining the implications of comparative advantage. Consider, for instance, these three textbooks: Lipsey and Chrystal (2004, ch. 33), Stiglitz and Walsh (2006, ch. 19) and Miller (2008, ch. 33). All of them discuss comparative advantage in the interpersonal case first: Lipsey and Chrystal (2004, p. 610) have a doctor who is a bad carpenter; Stiglitz and Walsh (2006, p. 427), the president of a company and her secretary; and Miller (2008, pp. 836), a creative advertising specialist and a computer artist. The analysis in the two first cases suggests that specialization is complete: the doctor does not devote time to be a carpenter, whereas the president brings in new clients and the secretary types letters. In Miller's case, complete specialization is clearly identified as the outcome: "the ad specialist specialized only in writing ad copy and the computer whiz specialized only in creating computer art renderings".

By dealing with comparative advantage in the interpersonal case before the international trade case, the three textbooks contribute to give the impression that there is no substantial difference between the conclusions obtained in each case. Two of them are explicit in this respect: after the analysis of the interpersonal case, Lipsey and Chrystal (2004, p. 610) hold that "The same principle also applies to nations"; and Stiglitz and Walsh (2006, p. 427) state that "The principle of comparative advantage applies to individuals as well as countries". Miller (2008, p. 834) simply says that "The best way to understand the gains from trade among nations is first to understand the output gains from specialization between individuals".

Consequently, being complete specialization the apparent outcome of the existence of comparative advantage in the interpersonal case, it appears that it should also be the outcome in the international trade case. But all the authors use "specialize" ambiguously. Miller (2008, p. 838) states, in a model involving the United States, India and the production of software and personal computers, that "U.S. residents will specialize in the activity in which they have a comparative advantage". Since, in

Miller's (2008, pp. 837–840) analysis, the alternative to autarky is complete specialization, one is led to believe that specialization is complete.

Stiglitz and Walsh (2006, p. 426) hold that “individuals and countries specialize in those goods in whose production they are relatively, not absolutely, most efficient”. Stiglitz and Walsh's (2006, pp. 428–429) example, which involves China, the United States, textiles and airplanes, hints at incomplete specialization. The authors confirm this impression by stating that comparative advantage may not lead to complete specialization. But incomplete specialization is artificially introduced by redefining commodities: what previously was the commodity “textiles” later becomes two commodities, “inexpensive textiles” and “higher-quality textiles”. This redefinition just begs the question: can a country specialize in inexpensive textiles incompletely?

Lipsey and Chrystal (2004, p. 613) add a qualification in their specialization result: “If trade is possible at some terms of trade between the two countries' opportunity costs of production, each country will specialize in the production of the good in which it has a comparative advantage”. Lipsey and Chrystal's (2004, p. 611) Table 33.2 explains the gains from specialization by considering incomplete specialization, because countries produce a little more of some commodity but do not abandon the production of the other commodity. This points to interpreting “specialize” as “partially specialize”, which is not consistent with their usage of “specialize” in the interpersonal case, which points to “completely specialize”.

One may contend that having to clarify whether specialization is complete or not is a minor point, as comparative advantage ensures that some partial specialization is always enough for countries to have incentive to trade. That is a valid objection. But then, on the one hand, why insisting in dealing first with the interpersonal case and suggesting a solution (complete specialization) that is not the intended solution for the international trade case? And, on the other hand, precision is always valuable: presenting partial specialization as the solution identifies the medicine but not the dose; presenting complete specialization as the solution identifies both the medicine and the dose.

The source of the ambiguity in the textbook explanation of the theory of comparative advantage probably lies in the fact that it abstracts from the countries' initial position. When that initial position is made explicit, it becomes obvious that having both countries initially producing together more of commodity *X* than what the country having comparative advantage in the production of *X* could produce implies that

specialization cannot be the solution. For a similar reason, if a big country faces a small country, it is difficult to justify that complete specialization will always be the solution.

Motivated by those ambiguities, this paper aims to provide an answer to the question of how likely it is for a country to choose to completely specialize when the alternative is autarky, which is the pair of alternatives considered in some examples presenting the theory of comparative advantage, such as those in Wikipedia (2010) and Miller (2008, pp. 838–839). Why is this restriction to two alternatives worth considering?

On the one hand, it is conceivable that these two could be the relevant alternatives when some trade agreement between two countries is being negotiated, as one of the two countries may demand that both countries completely specialize for the trade agreement to be acceptable. This condition might be reasonable if comparative advantage would lead only one country  $c$  to completely specialize, because in that case  $c$  would assume the risky position of having the supply of some commodity fully depend on another country  $c'$ . In these circumstances,  $c$  could invoke reciprocity when asking  $c'$  to absolutely rely on  $c$  for the supply of the other commodity. Autarky and complete specialization may also be the basic options when the economic integration between two regions of the same country is the result of a political decision: in a country, it is rarely the case that everything is produced everywhere.

On the other hand, from a conceptual point of view, it might be worth comparing the two extreme situations. In fact, autarky is represented by just one possibility (no amount of any commodity exchanged with another country), while openness consists of an infinite set of possibilities (the smallest amount of some commodity exchanged with another country makes autarky the defeated option). It is therefore a priori easy for openness to become the winning alternative, despite the fact that many exchange situations are closer to autarky than to full openness. One way of removing this pro trade bias is to face autarky with just the complete opposite: full openness arising from complete specialization.

The analysis is carried out in the standard Ricardian model (the one with two countries, two commodities, one input and linear production possibilities frontiers) by adding two assumptions. First, any point on a country's production possibilities frontier is equally likely to represent the initial situation of the country. And second, given the initial situation of each country, complete specialization occurs if, by adopting it, both countries can obtain, with respect to their initial positions, more of some commodity and at least as much of the other.

It is shown that these two assumptions make autarky a more likely choice than complete specialization (and, in the present context, trade). Accordingly, if in the Ricardian model countries are forced to choose between autarky (full closedness) and complete specialization (full openness), the more likely outcome is autarky.

## 2. Model

There are two countries, 1 and 2; two commodities,  $X$  and  $Y$ ; and the only input is labour,  $L$ . Country 1 is assumed to be endowed with the amount  $L_1 > 0$  of labour, whereas country 2 is assumed to be endowed with the amount  $L_2 > 0$ . In each country, all the available labour is applied to the production of some commodity. Though this is the usual assumption, it need not be presumed that  $X$  and  $Y$  are the only commodities. A more general interpretation is that, for country  $i \in \{1, 2\}$ , the amount of labour  $L_i$  can only be applied to the production of commodities  $X$  and  $Y$ . In this context, autarky is relative to  $X$  and  $Y$ .

In country 1,  $Y_1 = a_1 L_{1Y}$  is the production function of commodity  $Y$  and  $X_1 = b_1 L_{1X}$  is the production function of commodity  $X$ , where: (i)  $a_1$  and  $b_1$  are positive real numbers representing both the marginal and the average productivity of labour; and (ii) for  $\alpha \in \{X, Y\}$ ,  $L_{1\alpha}$  is the amount of labour in country 1 assigned to the production of commodity  $\alpha$ . Similarly, for country 2,  $Y_2 = a_2 L_{2Y}$  and  $X_2 = b_2 L_{2X}$  are the production functions of, respectively, commodity  $Y$  and commodity  $X$ , with  $a_2$  and  $b_2$  being positive real numbers.

The production of each commodity in a country is represented by a point of the country's production possibilities frontier. These frontiers are

$$Y_1 = a_1 L_1 - \frac{a_1}{b_1} X_1 \quad (1)$$

for country 1 and

$$Y_2 = a_2 L_2 - \frac{a_2}{b_2} X_2 \quad (2)$$

for country 2.

Without loss of generality, country 1 is supposed to have comparative advantage in the production of commodity  $X$ , so  $\frac{a_1}{b_1} < \frac{a_2}{b_2}$  and, hence, country 2 has comparative advantage in the production of commodity  $Y$ .

It is assumed that countries choose autarky or complete specialization. This assumption allows the representation of production possibilities frontiers in an Edgeworth box; see Fig. 1, drawn assuming  $\frac{a_1}{b_1} < \frac{a_2}{b_2}$ ,  $a_1L_1 < a_2L_2$  and  $b_1L_1 > b_2L_2$ . For any two points  $p$  and  $q$  on an Edgeworth box, define  $pq$  to be the distance between the two points.

Given the initial positions of the two countries, complete specialization (and, therefore, trade) is defined to occur if and only if the rectangle whose lower left corner is country 1's initial position has a non-empty intersection with the rectangle whose upper right corner is country 2's initial position. For instance, in Fig. 1,  $g$  represents the initial position of country 1 and  $h$  the initial position of country 2. In this case, complete specialization occurs because the intersection of the two rectangles (the shaded area) is non-empty. In fact, when country 1 completely specializes in the production of  $X$  and 2 in the production of  $Y$ , any point in the shaded area can be reached through trade and in any such point both countries obtain more of some commodity and not less of the other.

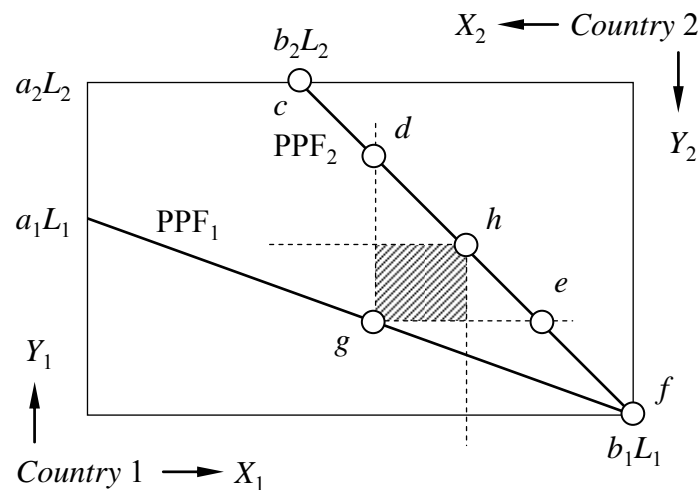


Fig. 1. Production possibilities frontiers in an Edgeworth box

In Fig. 1, complete specialization still occurs if, given  $g$ , the initial position of country 2 lies between points  $d$  and  $e$  (both included). Consequently, if no point along  $PPF_2$  is more likely to represent country 2's initial position, the probability that complete specialization occurs when  $g$  is country's 1 initial position is the distance  $de$  divided by the distance  $cf$ : favourable cases divided by total number of cases. By averaging over all the points on  $PPF_1$ , a probability that complete specialization (and, therefore, trade) occurs can be defined. It will be shown that this probability is always smaller than  $\frac{1}{2}$ .

### 3. Analysis

Let (1) and (2) define the production possibilities frontiers of countries 1 and 2, with  $\frac{a_1}{b_1} < \frac{a_2}{b_2}$ . Set  $A = \frac{a_1}{a_2}$ ,  $B = \frac{b_1}{b_2}$  and  $L = \frac{L_1}{L_2}$ . The parameter  $A$  measures how productive country 1 is producing commodity  $Y$  in comparison with country 2. Analogously,  $B$  measures how productive country 1 is producing commodity  $X$  in comparison with country 2. In addition,  $L$  measures how large the labour force is in country 1 in comparison with that of country 2. Note that  $\frac{a_1}{b_1} < \frac{a_2}{b_2}$  amounts to  $A < B$ . There are three cases to consider.

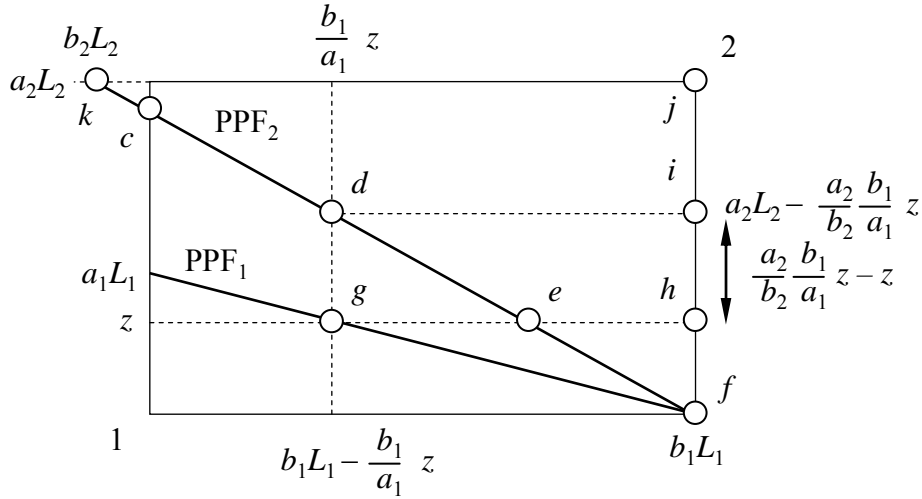


Fig. 2. Computing the probability of trading when  $a_1L_1 < a_2L_2$  and  $b_1L_1 \leq b_2L_2$  (Case 1)

Case 1:  $a_1L_1 < a_2L_2$  and  $b_1L_1 \leq b_2L_2$ ; that is,  $A < B \leq \frac{1}{L}$ . This case is represented in Fig.

2. For every point  $g$  on  $PPF_1$ , the probability of trading is  $\frac{de}{kf}$ , which equals  $\frac{ih}{jf}$ . Any such  $g$  can be associated with a unique value  $z \in [0, a_1L_1]$ . Given the  $z$  corresponding to  $g$ ,  $hf$  in Fig. 2 is  $z$  itself,  $ij$  is  $a_2L_2 - \frac{a_2}{b_2} \frac{b_1}{a_1} z$  and  $jf$  is  $a_2L_2$ . Therefore,  $ih$  is  $\left( \frac{a_2}{b_2} \frac{b_1}{a_1} - 1 \right) z$ .

In view of this, the probability  $p_1(z)$  of trading (when a point on  $PPF_1$  is chosen so that  $z$

is the amount produced of commodity  $Y$ ) is  $p_1(z) = \frac{ih}{jf} = \frac{\left( \frac{a_2}{b_2} \frac{b_1}{a_1} - 1 \right) z}{a_2L_2}$ . Fig. 3 shows the

graph of the function  $p_1(z)$  for  $z \in [0, a_1L_1]$ .

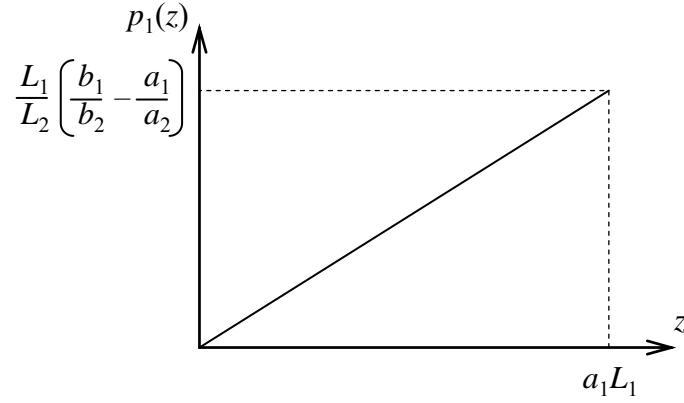


Fig. 3. The probability function of trading when  $a_1L_1 < a_2L_2$  and  $b_1L_1 \leq b_2L_2$  (Case 1)

Assuming that country 1 can choose any value of  $z$  with the same probability, the average probability of trading in Case 1 is  $p_1 = \frac{1}{2} \frac{L_1}{L_2} \left( \frac{b_1}{b_2} - \frac{a_1}{a_2} \right)$ , which is equivalent to

$$p_1 = \frac{1}{2} L(B - A). \quad (3)$$

Case 2:  $a_1L_1 < a_2L_2$  and  $b_1L_1 > b_2L_2$ ; that is,  $A < \frac{1}{L} < B$ . In this case, as shown in Fig. 4, PPF<sub>2</sub> intersects what for country 2 is the horizontal axis (at a point such as  $d$  in Fig. 4) before intersecting (at a point such as  $c$  in Fig. 2) what for country 1 is the vertical axis. This leads to two possibilities, depending on whether country 1 chooses a point above or below the point  $g$  identified in Fig. 4. At that point or below, the analysis is exactly as in Case 1. Above point  $g$ , the resulting probability is smaller than the one resulting in Case 1 because PPF<sub>2</sub> stops at  $d$ .

Case 2a: the point  $c$  chosen by country 1 is  $g$  or lies below  $g$  along PPF<sub>1</sub>. This occurs when  $0 \leq z \leq \frac{a_1}{b_1} b_2 L_2$ ; see Fig. 4. Following the analysis in Case 1, the probability of trading at  $c$  is  $\frac{je}{df}$ , which equals  $\frac{hk}{if}$ . Since  $hk$  is  $\left( \frac{a_2}{b_2} \frac{b_1}{a_1} - 1 \right) z$ , the probability  $p_2(z)$  of trading (when a point on PPF<sub>1</sub> is chosen so that  $z \leq \frac{a_1}{b_1} b_2 L_2$  is the amount produced of

$$\text{commodity } Y) \text{ is } p_2(z) = \frac{hk}{if} = \frac{\left( \frac{a_2}{b_2} \frac{b_1}{a_1} - 1 \right)}{a_2 L_2} z, \text{ with } 0 \leq z \leq \frac{a_1}{b_1} b_2 L_2.$$

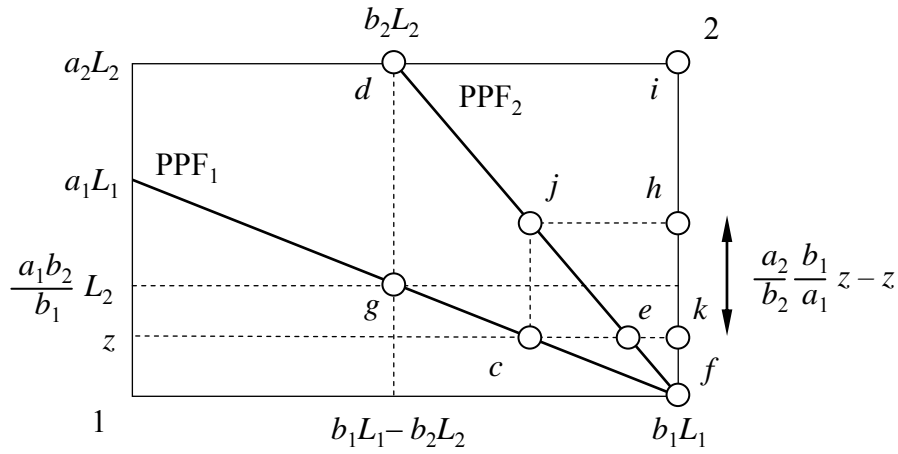


Fig. 4. Computing the probability of trading when  $a_1L_1 < a_2L_2$  and  $b_1L_1 > b_2L_2$  (Case 2a)

Case 2b: the point  $c$  chosen by country 1 is above  $g$  along  $PPF_1$ . This occurs when  $a_1L_1 \geq z > \frac{a_1}{b_1} b_2L_2$ ; see Fig. 5. Now, the probability of trading at  $c$  is  $\frac{de}{df}$ , which equals  $\frac{ih}{if}$ .

Since  $ih = a_2L_2 - z$ , the probability  $p_2(z)$  of trading (when a point in  $PPF_1$  is chosen so that  $z > \frac{a_1}{b_1} b_2L_2$  is the amount produced of commodity  $Y$ ) is  $p_2(z) = \frac{ih}{if} = \frac{a_2L_2 - z}{a_2L_2} = 1 -$

$\frac{z}{a_2L_2}$ , with  $a_1L_1 \geq z > \frac{a_1}{b_1} b_2L_2$ . Fig. 6 shows the graph of the function  $p_2(z)$  for  $z \in [0, a_1L_1]$ .

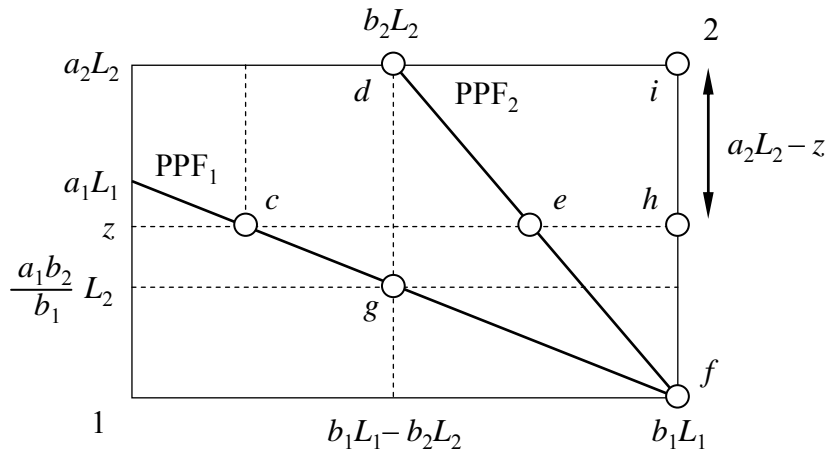


Fig. 5. Computing the probability of trading when  $a_1L_1 < a_2L_2$  and  $b_1L_1 > b_2L_2$  (Case 2b)

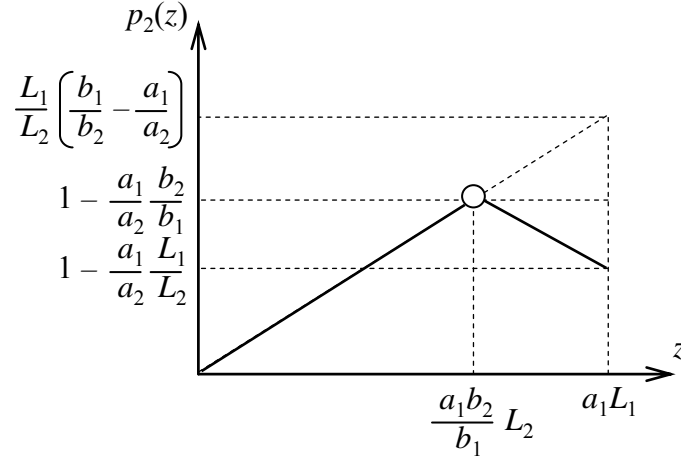


Fig. 6. The probability function of trading when  $a_1 L_1 < a_2 L_2$  and  $b_1 L_1 > b_2 L_2$  (Case 2)

Assuming that country 1 can choose any value of  $z$  with the same probability, the

probability of choosing  $z \leq \frac{a_1 b_2 L_2}{b_1}$  (so that Case 2a applies) is  $\frac{\frac{a_1 b_2 L_2}{b_1}}{a_1 L_1} = \frac{b_2 L_2}{b_1 L_1} = \frac{1}{BL}$ ,

in which case the average probability of trading is  $\frac{1}{2} \left( 1 - \frac{a_1 b_2}{a_2 b_1} \right)$ , which is equivalent to

$\frac{1}{2} \left( 1 - \frac{A}{B} \right)$ . On the other hand, the probability of choosing  $z > \frac{a_1 b_2 L_2}{b_1}$  (so Case 2b

applies) is  $1 - \frac{1}{BL}$ , making the average probability of trading to be equal to

$\frac{1}{2} \left( 1 - \frac{a_1 b_2}{a_2 b_1} - \left( 1 - \frac{a_1 L_1}{a_2 L_2} \right) \right)$ , which is equivalent to  $\frac{1}{2} A \left( L - \frac{1}{B} \right)$ . In sum, the average

probability of trading in case 2 is  $p_2 = \frac{1}{BL} \frac{1}{2} \left( 1 - \frac{A}{B} \right) + \left( 1 - \frac{1}{BL} \right) \frac{1}{2} A \left( L - \frac{1}{B} \right) =$

$\frac{1}{2} \left( AL + \frac{1}{BL} \right) - AB$ , which can be equivalently expressed as

$$p_2 = \frac{1}{2} A \left( L - \frac{1}{B} \right) + \frac{1}{2} \frac{1}{B} \left( \frac{1}{L} - A \right). \quad (4)$$

Case 3:  $a_1 L_1 \geq a_2 L_2$ ; that is,  $\frac{1}{L} \leq A < B$ . In this case, as shown in Fig. 7, PPF<sub>1</sub> intersects

the vertical axis of country 1 outside the Edgeworth box (the assumption  $A < B$  implies that PPF<sub>2</sub> cannot intersect, as in Case 1, the vertical axis of country 1, which means that  $b_1 L_1 > b_2 L_2$ ). This gives rise to three possibilities, depending on whether country 1 chooses a point on PPF<sub>1</sub> above the point  $g$  in Fig. 7, between  $g$  and  $e$ , or below  $e$ .

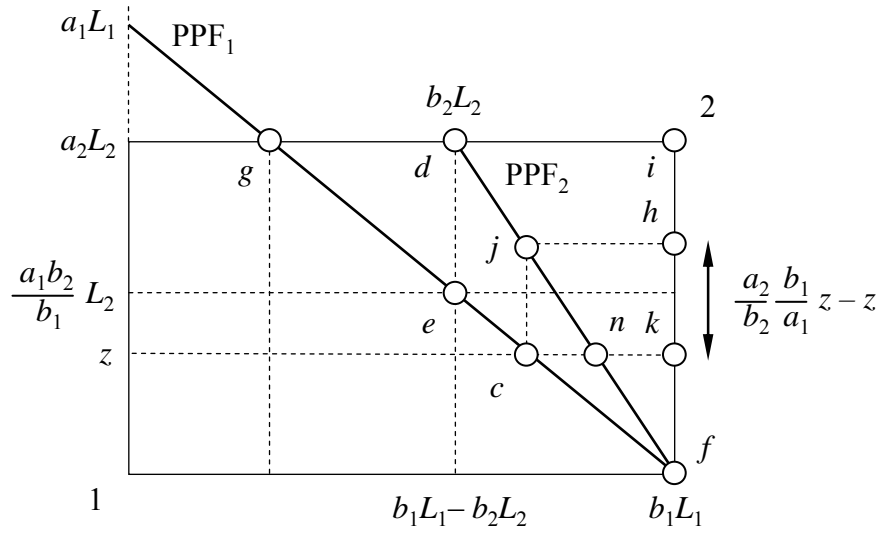


Fig. 7. Computing the probability of trading when  $a_1L_1 \geq a_2L_2$  (Case 3a)

Case 3a: the point  $c$  chosen by country 1 is  $e$  or lies below  $e$  along PPF<sub>1</sub>. This occurs for  $0 \leq z \leq \frac{a_1 b_2}{b_1} L_2$ ; see Fig. 7. The resulting case is analogous to Case 1 (and Case 2a). So

following the analysis in Case 1, the probability of trading at  $c$  in Fig. 7 is  $\frac{jn}{df}$ , which

equals  $\frac{hk}{if}$ . Since  $hk$  is  $\left(\frac{a_2 b_1}{b_2 a_1} - 1\right)z$ , the probability  $p_3(z)$  of trading (when a point on

PPF<sub>1</sub> is chosen so that  $z \leq \frac{a_1 b_2}{b_1} L_2$  is the amount produced of commodity  $Y$ ) is  $p_3(z) =$

$$\frac{hk}{if} = \frac{\left(\frac{a_2 b_1}{b_2 a_1} - 1\right)z}{a_2 L_2}, \text{ with } 0 \leq z \leq \frac{a_1 b_2}{b_1} L_2.$$

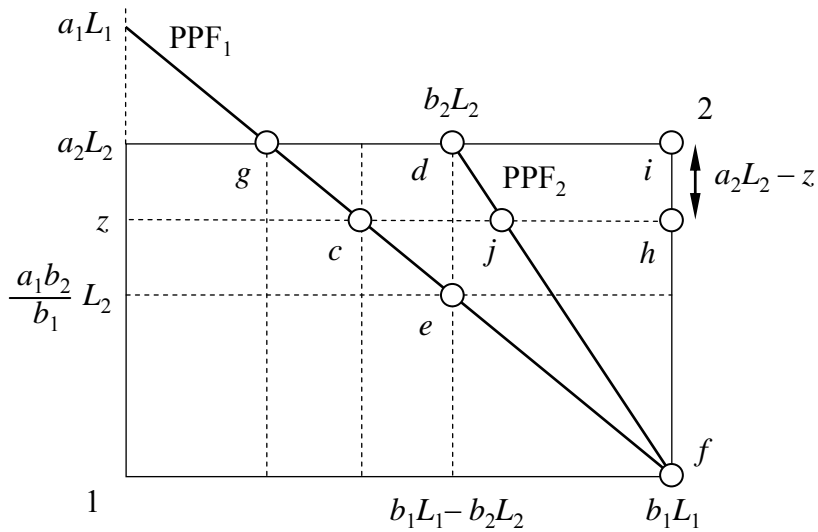


Fig. 8. Computing the probability of trading when  $a_1L_1 \geq a_2L_2$  (Case 3b)

Case 3b: the point  $c$  chosen by country 1 lies between  $g$  and  $e$  along PPF<sub>1</sub>. This occurs when  $a_2L_2 > z > \frac{a_1}{b_1}b_2L_2$ ; see Fig. 8. The resulting case is analogous to Case 2b. Thus, the probability of trading at  $c$  in Fig. 8 is  $\frac{dj}{df}$ , which equals  $\frac{ih}{if}$ . Since  $ih = a_2L_2 - z$ , the probability  $p_3(z)$  of trading (when a point on PPF<sub>1</sub> is chosen so that  $a_2L_2 > z > \frac{a_1}{b_1}b_2L_2$  is the amount produced of commodity  $Y$ ) is  $p_3(z) = \frac{ih}{if} = \frac{a_2L_2 - z}{a_2L_2} = 1 - \frac{z}{a_2L_2}$ , with  $a_2L_2 > z > \frac{a_1}{b_1}b_2L_2$ .

Case 3c: the point chosen by country 1 lies above  $g$  along PPF<sub>1</sub>. This occurs when  $a_1L_1 \geq z \geq a_2L_2$ . At any such point no trade occurs, because country 1 is already producing more commodity  $Y$  than country 2 can produce.

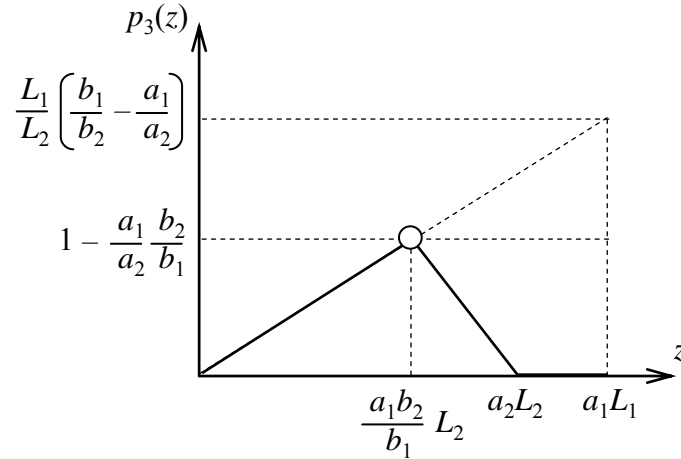


Fig. 9. The probability function of trading when  $a_1L_1 \geq a_2L_2$  (Case 3)

Fig. 9 shows the graph of the function  $p_3(z)$  for  $z \in [0, a_1L_1]$ , where  $p_3(z) = 0$  when  $z \in [a_2L_2, a_1L_1]$ . Assuming that country 1 can choose any value of  $z$  with the same likelihood, the average probability in Case 3a and Case 3b is  $\frac{1}{2} \left( 1 - \frac{a_1}{a_2} \frac{b_2}{b_1} \right) = \frac{1}{2} \left( 1 - \frac{A}{B} \right)$ . Given that the probability that  $z \in [0, a_2L_2]$  is  $\frac{a_2L_2}{a_1L_1} = \frac{1}{AL}$ , it follows that the average probability of trading in Case 3 is  $p_3 = \frac{1}{AL} \frac{1}{2} \left( 1 - \frac{A}{B} \right)$ , which can be equivalently expressed as

$$p_3 = \frac{1}{2} \left( \frac{1}{AL} - \frac{1}{BL} \right). \quad (5)$$

#### 4. Result

Table 10 summarizes the analysis carried out in Section 3. The first column identifies the three possible cases that can arise when  $A < B$  (the case  $A = B$  is not interesting and the case  $A > B$  is simply the case  $A < B$  with the names of the countries exchanged). The second column defines the conditions characterizing each case. The third column lists the results (3), (4) and (5), which establish the average probability of trading in each case. The fourth column shows, for each case, the necessary and sufficient condition for the probability of trading to be smaller than the probability of not trading.

Case	arises when	probability $p$ of trading	condition for $p < 1/2$
1	$0 < AL < BL \leq 1$	$\frac{1}{2}(BL - AL)$	$BL < 1 + AL$
2	$0 < AL < 1 < BL$	$\frac{1}{2}\left(AL + \frac{1}{BL}\right) - \frac{A}{B}$	$BL > \frac{1 - 2AL}{1 - AL}$
3	$1 \leq AL < BL$ or $0 < \frac{1}{BL} < \frac{1}{AL} \leq 1$	$\frac{1}{2}\left(\frac{1}{AL} - \frac{1}{BL}\right)$	$\frac{1}{BL} > \frac{1}{AL} - 1$

Table 10. Results and conditions for not trading to be more probable than trading

**Proposition 1.** Let (1) and (2) define the production possibilities frontiers of two countries, 1 and 2, which may produce only two commodities,  $X$  and  $Y$ , where  $a_1, a_2, b_1, b_2, L_1$  and  $L_2$  are positive constants such that  $\frac{a_1}{b_1} < \frac{a_2}{b_2}$  (so country 1 has comparative advantage in the production of  $X$  and country 2 has comparative advantage in the production of  $Y$ ). For point  $(X_1, Y_1)$  satisfying (1) and point  $(X_2, Y_2)$  satisfying (2), define trade to occur if and only if  $X_1 + X_2 < b_1L_1$  and  $Y_1 + Y_2 < a_2L_2$  (so specialization based on comparative advantage allows both countries to consume more of each commodity). Assume that, for each country, no point along the production possibilities frontier is more likely to be initially chosen than another point. Then the probability of not trading is always greater than the probability of trading.

*Proof.* The proof amounts to verifying that, for each of the three cases in Table 10, the inequality in the fourth column holds. In Case 1, the assumptions  $AL > 0$  and  $BL \leq 1$  imply  $BL < 1 + AL$ . In Case 2, since  $AL > 0$ ,  $\frac{1 - 2AL}{1 - AL}$  is always smaller than 1. As a

result, the assumption  $BL > 1$  yields  $BL > \frac{1-2AL}{1-AL}$ . In Case 3, the assumption  $\frac{1}{AL} \leq 1$  implies  $\frac{1}{AL} - 1 \leq 0$ , so the assumption  $\frac{1}{BL} > 0$  guarantees  $\frac{1}{BL} > \frac{1}{AL} - 1$ . ■

## 5. Comments

Proposition 1 incorporates two assumptions into the basic Ricardian model of trade. One refers to the initial situation of the countries: it is assumed that any point on a country's production possibilities frontier is equally likely to be the point where the decision to engage or not in trade has to be made. This is reasonable if, a priori, any social preference establishing the socially most preferred point on a production possibilities frontier is equally admissible.

The second assumption is perhaps more objectionable because it constraints to the utmost the possibility of trading: trade only occurs if both countries can get more of each commodity. This excludes the case in which the social preference is represented by the standard convex, decreasing and smooth indifference curves. This notwithstanding, the social preferences implicitly adopted in the paper (with L-shaped indifference curves) can be viewed as a limit case of the standard indifference curves. In this respect, (3), (4) and (5) specify lower bounds for the probability of trading in a more general model in which social preferences are simply assumed to be monotonic.

It is apparent that many types of social preferences will generate probabilities larger than those in (3), (4) and (5). But then the point would be that this increase in the probability of trading will depend on the specific type of preference considered. For instance, one can define social preferences so that, in Fig. 2, the two countries will trade if they start from  $c$  and  $g$ . But one can define social preferences leading to the opposite outcome. By contrast, it seems reasonable to presume that, under any admissible social preference, trade will occur in Fig. 1 when the countries are initially at  $g$  and  $h$ . It therefore makes sense to consider the question of how likely it is to trade by just paying attention to cases in which it seems to be beyond dispute that trade can occur.

This reasoning leads to a second justification that relies on the Pareto principle. In Fig. 1, with countries at  $c$  and  $g$ , the superiority of another situation reached through trade does not follow from the Pareto principle, because if there is trade between the two countries, the consumption of some commodity must be reduced in at least one country. As a consequence, some individual in some country must reduce his consumption of

some commodity. Hence, the superiority of the outcome of trade over the outcome of autarky depends on how individuals, first, and society, next, determine the way in which more consumption of some commodity compensates the loss of welfare due to the reduction in consumption of some other commodity. Since it is not obvious when one's consumption can be sacrificed for "the good of society", it is plausible to consider that "the good of society" may justify avoiding putting society in a situation in which such choices or judgements must be made. In that case, the only rule establishing when trade is unobjectionable is the Pareto principle: to have more of some commodity but not less of the other. So (3), (4) and (5) can be seen as measures of how likely it is to have Pareto improvements through trade.

Finally, (3), (4) and (5) are also useful to determine what changes in  $A$ ,  $B$  and  $L$  under autarky make trade subsequently more or less likely. Roughly speaking, Case 1 represents the situation in which country 1 is "small", because the maximum that country 1 could produce of each commodity is less than the maximum that country 2 could produce. Case 3 represents the reverse situation: country 2 is the "small" country. So, in a sense, Case 1 and Case 3 describe the same situation.

In fact, in both cases, the probability of trading decreases with  $A$  and increases with  $B$  and  $L$ :  $\frac{\partial p_1}{\partial A} < 0$ ,  $\frac{\partial p_3}{\partial A} < 0$ ,  $\frac{\partial p_1}{\partial B} > 0$ ,  $\frac{\partial p_3}{\partial B} > 0$ ,  $\frac{\partial p_1}{\partial L} > 0$  and  $\frac{\partial p_3}{\partial L} > 0$ . Considering Case 1, these results mean that the probability that trade occurs increases if country 1: (i) becomes comparatively less productive in the commodity in whose production the country does not have comparative advantage ( $A$  decreases); (ii) becomes comparatively more productive in the commodity in whose production the country has comparative advantage ( $B$  increases); or (iii) becomes comparatively more populated ( $L$  increases), thereby becoming closer in size to the "big" country 2.

Case 2 represents the intermediate situation in which each country is "big" in the industry producing the commodity in whose production the country has comparative advantage. Case 2 seems to be the one in which it is intuitively more likely for trade to occur, because a big country does not seem to be in need to trade with a small country (Case 1 and Case 3). Unexpectedly, the apparently intuitive results from Case 1 and Case 3 fail:  $\frac{\partial p_2}{\partial A} = \frac{L}{2} - \frac{1}{B}$ ,  $\frac{\partial p_2}{\partial B} = \frac{1}{B^2} \left( A - \frac{1}{2L} \right)$  and  $\frac{\partial p_2}{\partial L} = \frac{1}{2} \left( A - \frac{1}{BL^2} \right)$ , so all the derivatives can be positive or negative. It is therefore possible to have  $\frac{\partial p_2}{\partial A} > 0$ , which occurs when  $BL > 2$ ; or to have  $\frac{\partial p_2}{\partial B} < 0$ , which occurs when  $AL < \frac{1}{2}$ ; or to have  $\frac{\partial p_2}{\partial L}$

$< 0$ , which occurs when  $(AL)(BL) < 1$ . It is even possible for the three results to hold simultaneously (for instance, with  $BL = 2.1$  and  $AL = 0.4$ ), in which case the probability of trading decreases when country 1 becomes, at the same, smaller and more (less) productive in the commodity in whose production the country has (does not have) comparative advantage.

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