Competitive markets as two-player games

Antonio Quesada[†]

Departament d'Economia, Universitat Rovira i Virgili, Avinguda de la Universitat 1, 43204 Reus, Spain

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Abstract
With the aim of providing a simple game-theoretic justification of market equilibrium, competitive markets having a unique market equilibrium, a decreasing market demand function and an increasing linear market supply function are considered. With each such market M, a two-player simultaneous game G_M is associated. In G_M, a player represents producers and the other player represents consumers. The players' aim is to maximize the surplus of the group they represent. The market equilibrium of M is shown to correspond to the only strategy profile in G_M that survives the iterated deletion of weakly dominated strategies. This result does not hold for the perfect information version of G_M in which the player representing consumers moves first.

Keywords: Competitive market; Market equilibrium; Nash equilibrium; Non-cooperative game; Weakly dominated strategy.

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[†] E-mail address: aqa@urv.cat.

1. Introduction

Monopoly, duopoly and perfect competition are, probably, the basic market models. Already in 1838, Cournot (1927) studied the three models mathematically. His analysis suggests that, conceptually, the cornerstone model is duopoly: reaction functions yield the monopoly solution when one of the duopolists leaves the market; and the competitive solution emerges by replicating many times the duopoly to the point that the production of any given producer has an inappreciable effect on the market price.

Since the Cournot duopoly can be formally represented as a two-player non-cooperative game, there arises the question of whether these three theoretical market structures can be reduced to two-player games. The possibility of such reduction is at least interesting from a pedagogical point of view because one could then present these three fundamental models to students as elaborations of the simplest type of non-trivial non-cooperative game.

In monopoly, (Cournot) duopoly and perfect competition consumers are represented by a market demand function. In this respect, consumers can be viewed as a single player whose best replies to the market price are captured by the market demand function. With this interpretation, monopoly can be described as a two-player game in which the monopolist chooses the market price and the quantity supplied first and next consumers choose the total quantity demanded. If consumers aim at maximizing their surplus and the monopolist tries to maximize his own surplus then the solution of this game, obtained by backward induction, coincides with the monopoly solution.

Duopoly could also be described in terms of two-player games. This time there are two such games. In game 1, the duopolists play a simultaneous game against each other and the Cournot equilibrium provides the solution to this game. In game 2, producers play against consumers. Producers choose the market price and the total quantity supplied and consumers choose the total quantity demanded once they know the market price. Game 2 would be just like the monopoly game, so duopoly can be seen as a monopoly in which there is a first stage at which the total quantity supplied is determined. When each of the three players aims at maximizing his own surplus, the solution of the whole game is also obtained by backward induction: knowing the reaction of consumers in game 2, the Cournot equilibrium is determined in game 1; given the total amount q supplied by duopolists in the Cournot equilibrium, they choose in game 2 as the market price the maximum price that guarantees that q will be sold; and, given that price, consumers choose q. The result is nothing else but the solution of the Cournot duopoly.

This note is motivated by the idea of pursuing the reduction to two-player games further to include perfect competition. A successful reduction in those terms would provide a simple game-theoretic justification of the competitive market equilibrium concept; for a more sophisticated approach based on dynamic matching and bargaining games, see Gale (2000).

Textbooks justify market equilibrium in terms of "market forces". For instance, Pindyck and Rubinfeld (1995, p. 19) refer to the "market mechanism" as a tendency to reach the market equilibrium; Lipsey and Chrystal (2004, p. 50) speak of a "driving force", acting on the market price, that they call the law of price adjustment; and Marshall (1920, p. 287) explains how the market equilibrium quantity is attained in terms of an "active force" tending to increase or decrease the quantity supplied.

The justification suggested here (Proposition 3.4) establishes a correspondence between the only equilibrium of a competitive market M (with linear market supply function) and the only undominated Nash (1951) equilibrium of a simultaneous two-player game G_M associated with M. In G_M, one of the players (player s) represents and plays on behalf of producers and the other player (player d) represents and plays on behalf of consumers. Whereas s chooses the market price with the aim of maximizing the producers' surplus, d chooses the quantity demanded with the aim of maximizing the consumers' surplus.

This model can be roughly justified as follows. The treatment of consumers is consistent with their treatment in the two-player games associated with monopoly and duopoly: the market demand function essentially describes the player associated with consumers. Hence, by symmetry, producers could then be treated as a single player, this time associated with the market supply function. Consistency with the representation of producers in monopoly and duopoly seems to require that player s should choose both the market price and the total quantity supplied. But to reinforce the symmetric treatment of producers and consumers, it can be assumed that the producers' choice of the market price determines the quantity supplied: when p is the market price chosen, producers supply the quantity that the market supply function assigns to p. In view of this, it may be simply assumed that producers choose the market price.

To test the robustness of Proposition 3.4 with respect to the game structure, another justification of market equilibrium (Proposition 3.6) is provided when a sequential two-player perfect information game in which s moves first is associated with M. In this case, subgame perfection justifies market equilibrium.

On the negative side, Proposition 3.6 does not hold for the game in which d chooses first and Remark 3.5 shows that Proposition 3.4 need not hold when the market supply function is not linear. Despite this, Proposition 3.4 may be useful to justify market equilibrium in the basic case in which the two market functions are linear.

A normative justification of market equilibrium asserts that the sum of the producers' and the consumers' surplus is maximized at the market equilibrium; see Besanko and Braeutigam (2002, pp. 411-413). Proposition 3.4 improves this justification, as the joint surplus is maximized by letting each side of the market maximize its own surplus independently. This result reinforces the invisible hand motto.

2. Definitions

Fix a commodity C and let R designate the set of non-negative real numbers.

Definition 2.1. A market supply function for C is a function $q^s : R \to R$: (i) mapping price $p \in R$ of C into the total quantity $q^s(p) \in R$ of C supplied at price p; (ii) non-decreasing on R; and (iii) for some $p' \geq 0$, strictly increasing on $\{p \in R: p \geq p'\}$ and equal to zero on $\{p \in R: 0 \leq p \leq p'\}$.

Definition 2.2. A market demand function for C is a function $q^d : R \to R$: (i) mapping price $p \in R$ of C into the total quantity $q^d(p) \in R$ of C demanded at price p; (ii) non-increasing on R; and (iii) either strictly decreasing on R or, for some $p' \geq 0$, strictly decreasing on $\{p \in R: 0 \leq p \leq p'\}$ and equal to zero on $\{p \in R: p \geq p'\}$.

Definition 2.3. A competitive market M for C is a pair (q^s, q^d) consisting of a market supply function q^s and a market demand function q^d.

As customary, the graphical representation of the market functions of a competitive market will have the price value represented on the y-axis and the quantity value on the x-axis. For price p, let $l_x(p)$ designate the line that is parallel to the x-axis at value p and, for quantity q, let $l_y(q)$ designate the line that is parallel to the y-axis at value q.

Definition 2.4. For price p and quantity demanded q in $M = (q^s, q^d)$, the producers' surplus $\lceil p, q \rceil$ is the total revenue $p \cdot min\{q^s(p), q\}$ minus the variable cost of producing the amount $q^s(p)$, cost which is defined to be the area below the market supply function, above the x-axis, to the right of the y-axis and to the left of $l_y(q^s(p))$.

Definition 2.5. For price p and quantity demanded q in $M = (q^s, q^d)$, the consumers' surplus $\langle p, q\rangle$ is defined as follows. If $q \le q^d(p)$ then $\langle p, q\rangle$ is the area below the market demand function, above $l_x(p)$, to the right of the y-axis and to the left of $l_y(q)$. If $q > q^d(p)$ then $\langle p, q\rangle$ is $\langle p, q^d(p)\rangle$ minus the area above the market demand function, below $l_x(p)$, to the right of $l_y(q^d(p))$ and to the left of $l_y(q)$.

Definition 2.6. For competitive market $M = (q^s, q^d)$ of C, the simultaneous game $G_M = (\{s, d\}, \{R, Q\}, \{u_s, u_d\})$ associated with M is the two-player game such that: (i) s is a player representing producers and d a player representing consumers; (ii) the strategy set of player s is R, whose members are interpreted as prices; (iii) the strategy set of player d is $Q := R$ if $q^d(0) \neq 0$ and $Q := [0, q^d(0)]$ otherwise, with each member of Q interpreted as a quantity demanded; (iv) a strategy profile (p, q) in G_M is a member of $R \times Q$ establishing the market price p chosen by s and the total quantity demanded q chosen by d; and (v) the real-valued payoff functions u_s and u_d associate with each strategy profile (p, q) the values $u_s(p, q) := \lceil p, q\rceil$ and $u_d(p, q) := \langle p, q\rangle$.

In game G_M, player s chooses a price p in ignorance of the choice made by d, who chooses, ignoring the price chosen by s, the quantity q that consumers are willing to buy. Once p is chosen, the agents represented by the market supply function supply the amount $q^s(p)$. For the choices p and q, the payoff of player s is given by the producers' surplus $\lceil p, q\rceil$, whereas the payoff of d is given by the consumers' surplus $\langle p, q\rangle$.

Definition 2.7. For $q \in Q$, $p \in R$ is a best reply to q in G_M if, for all $p' \in R$, $\lceil p, q\rceil \ge \lceil p', q\rceil$. For $p \in R$, $q \in Q$ is a best reply to p in G_M if, for all $q' \in Q$, $\langle p, q\rangle \ge \langle p, q'\rangle$.

Definition 2.8. Strategy p is weakly dominated (WD) in a game G obtained from G_M by removing a certain set of strategies from the players' strategy sets if there is a strategy p' in G such that, for every strategy q in G, $u_s(p', q) \ge u_s(p, q)$ and, for some strategy q in G, $u_s(p', q) > u_s(p, q)$. That strategy q is weakly dominated is analogously defined.

Definition 2.9. Strategy profile (p, q) of G is a Nash equilibrium (NE) if, for every strategy p' in G and every strategy q' in G, $u_s(p, q) \ge u_s(p', q)$ and $u_d(p, q) \ge u_d(p, q')$.

Definition 2.10. For competitive market $M = (q^s, q^d)$ of C, the sequential game ${}_s\Gamma_M$ associated with M is the two-player perfect information game (for a definition, see Myerson (1991)) in which s chooses a price p first and, knowing this choice, d ends the game by choosing a quantity demanded q, with the players' payoffs being as in G_M, that is, $u_s(p, q) := \lceil p, q\rceil$ and $u_d(p, q) := \langle p, q\rangle$.

Definition 2.11. For competitive market $M = (q^s, q^d)$ of C, the sequential game ${}_d\Gamma_M$ associated with M is the two-player perfect information game in which d chooses a quantity demanded q first and, knowing this choice, s ends the game by choosing a price p, with the players' payoffs being as in G_M.

Definition 2.12. A strategy profile in ${}_s\Gamma_M$ (${}_d\Gamma_M$) is a subgame perfect equilibrium (SPE) if, for every node assigned to player s (d), the choice that the profile associates with that node is a best reply at that node and the choice that the profile associates with the root is a best reply for player s (d) to the choices associated with the rest of nodes.

3. Results

In the following results, M is a competitive market with a linear market supply function and a unique market equilibrium. By Lemma 3.1, the market supply function determines the set $\{p \in R: q^s(p) = q\}$ of best replies to q in G_M of the player representing producers.

Lemma 3.1. Let $M = (q^s, q^d)$ be a competitive market satisfying (1) and (2). For $q > 0$, define $p^s(q) := (q + d)/c$. Then: (i) for all $q \in Q\backslash\{0\}$ and $p \in R\backslash\{p^s(q)\}$, $\lceil p^s(q), q\rceil > \lceil p, q\rceil$; and (ii) for all $q \in Q$, $p \in [0, d/c]$ and $p' > d/c$, $0 = \lceil p, q\rceil > \lceil p', 0\rceil$.

The market supply function in M satisfies, for some $c > 0$ and $d \geq 0$,

$$q^s(p) := cp - d \text{ if } p > d/c \text{ and } q^s := 0 \text{ if } p \in [0, d/c]. \tag{1}$$

$$M \text{ has a unique market equilibrium } (p^*, q^*), \text{ with } p^* > 0 \text{ and } q^* > 0. \tag{2}$$

Proof. (i) Let $q > 0$, so $p' := p^s(q) > d/c$ and $\lceil p', q\rceil = (p' - d/c)q/2$. Let $p \in R\backslash\{p'\}$. Case 1: $p < p'$. If $p < d/c$ then $\lceil p, q\rceil = 0 < \lceil p', q\rceil$. If $p \geq d/c$ then, as $q > q^s(p)$ and $p' > p$, $\lceil p, q\rceil = (p - d/c)q^s(p)/2 < (p' - d/c)q/2 = \lceil p', q\rceil$. Case 2: $p > p'$. Let $\Delta p := p - p'$ and $\Delta q := q^s(p) - q$. In passing from (p', q) to (p, q), the gain in the producers' surplus is $\Delta p{\cdot}q$, whereas the loss is $\Delta q{\cdot}p' + \Delta p\Delta q/2$. Hence, $\lceil p', q\rceil > \lceil p, q\rceil \Leftrightarrow \Delta p{\cdot}q < \Delta q{\cdot}p' + \Delta p\Delta q/2$. With $a = (p', q)$, $b = (p, q^s(p))$ and the price elasticity of supply from a to b being $\varepsilon^s_{ab} := \Delta q{\cdot}p'/\Delta p{\cdot}q$, it follows that $\lceil p', q\rceil > \lceil p, q\rceil \Leftrightarrow \varepsilon^s_{ab} > 1 - \Delta q/2q$. By (1), $\varepsilon^s_{ab} = cp' / (cp' - d)$. Thus, $\varepsilon^s_{ab} \geq 1$, $\varepsilon^s_{ab} > 1 - \Delta q/2q$ and $\lceil p', q\rceil > \lceil p, q\rceil$. (ii) If $p \in [0, d/c]$ then, as $q^s(p) = 0$, for all $q \in R$, $\lceil p, q\rceil = 0$. If $p > d/c$, as $min\{q^s(p), 0\} = 0$ and $q^s(p) > 0$, $\lceil p, 0\rceil < 0$.■

By Lemma 3.2, the market demand function determines the best reply in G_M of the player d representing consumers for prices above the equilibrium price; and, for prices not above, any quantity not smaller than the quantity supplied is a best reply for d.

Lemma 3.2. Let $M = (q^s, q^d)$ be a competitive market satisfying (1) and (2). Then: (i) for all $p \geq p^*$ and $q \in Q\backslash\{q^d(p)\}$, $\langle p, q^d(p)\rangle > \langle p, q\rangle$; and (ii) for all $p \leq p^*$, $q < q^s(p)$ and $q' > q^s(p)$, $\langle p, q^s(p)\rangle = \langle p, q'\rangle > \langle p, q\rangle$.

Proof. (i) Let $p \geq p^*$ and $q \in Q\backslash\{q^d(p)\}$, so $q^s(p) \geq q^d(p)$. If $q < q^d(p)$, each unit between q and $q^d(p)$ yields consumers a positive surplus, so $\langle p, q^d(p)\rangle > \langle p, q\rangle$. If $q > q^d(p)$, each unit beyond $q^d(p)$ yields consumers a negative surplus, so $\langle p, q^d(p)\rangle > \langle p, q\rangle$. (ii) Let $p \leq p^*$. If $q < q^s(p)$ then every unit between q and $q^s(p)$ yields consumers a positive surplus, so $\langle p, q^s(p)\rangle > \langle p, q\rangle$. If $q > q^s(p)$ then, since $min\{q, q^s(p)\} = q^s(p)$, $\langle p, q^s(p)\rangle = \langle p, q\rangle$.■

Proposition 3.3. Let $M = (q^s, q^d)$ be a competitive market satisfying (1) and (2). Then the set of Nash equilibria of G_M is $\{(p, q) \in R \times Q: p \leq p^* \text{ and } q = q^s(p)\}$.

Proof. Step 1: there is no NE (p, q) of G_M with $p > p^*$. Let $p > p^*$, so $p > d/c$. By Lemma 3.2(i), $q^d(p)$ is the only best reply to p in G_M. If $q^d(p) > 0$ then, by Lemma 3.1(i), $p^s(q) < p$ is the only best reply to $q^d(p)$ in G_M. If $q^d(p) = 0$ then, by Lemma 3.1(ii), $[0, d/c]$ is the set of best replies to $q^d(p)$ in G_M. Step 2: there is no NE (p, q) of G_M with $p \leq p^*$ and $q < q^s(p)$. This follows from Lemma 3.2(ii). Step 3: there is no NE (p, q) of G_M with $p \leq p^*$ and $q > q^s(p)$. By Lemma 3.1(i), $p^s(q) > p$ is the only best reply to q. Step 4: for $p \leq p^*$, $(p, q^s(p))$ is an NE of G_M. Case 1: $d/c < p \leq p^*$. By Lemma 3.1(i), p is the only best reply to $q^s(p)$. By Lemma 3.2(ii), $q^s(p)$ is a best reply to p. Case 2: $0 \leq p \leq d/c$. In this case, $q^s(p) = 0$. By Lemma 3.1(ii), p is a best reply to $q = 0$. By Lemma 3.2(ii), $q = 0$ is a best reply to p.■

By Proposition 3.3, the set of Nash equilibria of G_M is given by the part of the graph of q^s that is not above the market equilibrium (p^*, q^*). Proposition 3.4 next states that the iterated removal of weakly dominated strategies in G_M leads to a game whose unique Nash equilibrium is the market equilibrium. This result does not depend on the order in which strategies are removed, since only one player has strategies removed at each step.

Proposition 3.4. Let $M = (q^s, q^d)$ be a competitive market satisfying (1) and (2). With $W_0(G_M) = G_M$, define, for $t \in \{1, 2, 3, 4\}$, $W_t(G_M)$ to be the game obtained from $W_{t-1}(G_M)$ by removing all the weakly dominated strategies. Then the only market equilibrium (p^*, q^*) of M is the only strategy profile in $W_4(G_M)$.

Proof. Step 1: $W_1(G_M)$ is obtained from G_M by removing all $q > q^*$. Step 1a: Player s has no WD strategy in G_M. This follows from Lemma 3.1. Step 1b: All the strategies in $\{q \in Q: q > q^*\}$ are WD by q^* in G_M. Let $q > q^*$. Case 1: $p > p^*$. Then $q^s(p) > q^*$ and $min\{q, q^s(p)\} > q^*$. As the consumers' surplus obtained from each unit beyond q^* is negative, $\langle p, q^*\rangle > \langle p, q\rangle$. Case 2: $p \leq p^*$. Then $q^s(p) \leq q^*$, so $min\{q, q^s(p)\} = q^s(p) < q^*$. As a result, for all $p \leq p^*$, $\langle p, q^*\rangle = \langle p, min\{q^*, q^s(p)\}\rangle = \langle p, min\{q, q^s(p)\}\rangle = \langle p, q\rangle$. Step 1c: The set of WD strategies of player d in G_M is $\{q \in Q: q > q^*\}$. By Step 1b, it suffices to show that no $q \leq q^*$ is WD in G_M. By Lemma 3.2(i), for every $q \leq q^*$, there is $p \geq p^*$ such that q is the only best reply to p. This proves that no $q \leq q^*$ is WD.

Step 2: $W_2(G_M)$ is obtained from $W_1(G_M)$ by removing all $p > p^*$. Step 2a: every $p > p^*$ is WD by p^* in $W_1(G_M)$. It suffices to show that, for all $p > p^*$ and $q < q^*$, $\lceil p, q^*\rceil > \lceil p, q\rceil$. The proof is analogous to Case 2 in the proof of Lemma 3.1. With $p > p^*$ and $q < q^*$, define $\Delta p := p - p^*$ and $\Delta q^s := q^s(p) - q^*$. Then, in passing from (p, q) to (p^*, q), the producers' surplus is reduced by the amount $\Delta p \cdot q < \Delta p \cdot q^*$, whereas it is increased by the amount $\Delta q^s \cdot p^* + \Delta q^s \Delta p/2$. It is then sufficient for $\lceil p^*, q\rceil > \lceil p, q\rceil$ that $\Delta p \cdot q^* < \Delta q^s \cdot p^* + \Delta q^s \Delta p/2$. With $b = (p^*, q^*)$, $c = (p, q^s(p))$ and $\varepsilon^s_{bc} = \Delta q^s \cdot p^*/\Delta p \cdot q^*$ being the price elasticity of supply from b to c, it is enough for $\lceil p^*, q\rceil > \lceil p, q\rceil$ that $\varepsilon^s_{bc} > 1 - \Delta q^s/2q^*$. By (1), $\varepsilon^s_{bc} = cp^* / (cp^* - d)$. Thus, $\varepsilon^s_{bc} \geq 1$ and $\varepsilon^s_{bc} > 1 - \Delta q^s/2q^*$. Step 2b: for all $p \leq p^*$, p is not WD in $W_1(G_M)$. By Lemma 3.1(i), for every $p > d/c$, there is $q \leq q^*$ such that p is the only best reply to q. And, by Lemma 3.1(ii), no $p \in [0, d/c]$ is WD.

Step 3: $W_3(G_M)$ is obtained from $W_2(G_M)$ by removing all $q < q^*$. By Lemma 3.2(ii), for all $p \leq p^*$ and $q < q^*$, $\langle p, q^*\rangle \geq \langle p, q\rangle$ and, by Lemma 3.2(i), for all $q < q^*$, $\langle p^*, q^*\rangle > \langle p^*, q\rangle$. Thus, every $q < q^*$ is WD by q^* in $W_2(G_M)$. Finally, the reasoning in Step 2b shows that no p in $W_2(G_M)$ is WD. Step 4: (p^*, q^*) is the only strategy profile in $W_4(G_M)$. By Step 3, it suffices to show that every $p < p^*$ is WD by p^* in $W_3(G_M)$. As q^* is player d's only strategy in $W_3(G_M)$, by Lemma 3.1(i), p^* is the only best reply to q^*.■

Remark 3.5. Propositions 3.3 and 3.4 need not hold when the market supply function is not linear, even if the market equilibrium is unique. To see this, let $M = (q^s, q^d)$ be such that: (i) $q^s(p) = 3p$, if $0 \leq p \leq 1$; (ii) $q^s(p) = p/3 + 8/3$, if $p > 1$; (iii) $q^d(p) = 0$, if $p > 5$; (iv) $q^d(p) = 10 - 2p$, if $5 \leq p < 4$; and (v) $q^d(p) = 10/3 - p/3$, if $4 \leq p \leq 0$. The only market equilibrium in M is $(p^*, q^*) = (1, 3)$ but $(p, q) = (4, 2)$ is an equilibrium in G_M surviving the iterated deletion of weakly dominated strategies. Notice that the producers selling quantity $q = 2$ have enough surplus to compensate the producers for the surplus lost from the amount $q^* - q$ that would be sold at $p = p^*$ but that is not sold when $p = 4$.

Proposition 3.6. Let $M = (q^s, q^d)$ be a competitive market satisfying (1) and (2). Then every subgame perfect equilibrium of ${}_s\Gamma_M$ prescribes p^* at the root of ${}_s\Gamma_M$ and prescribes $q \geq q^*$ at the node reached when p^* is played at the root.

Proof. Let r be the root of ${}_s\Gamma_M$. Step 1: no SPE prescribes $p > p^*$ at r. By Lemma 3.2(i), for all $p > p^*$, $q^d(p)$ is the only best reply to p. If $q^d(p) > 0$ then, by Lemma 3.1(i), the best reply to $q^d(p)$ is some $p < p^*$. If $q^d(p) = 0$ then $\lceil p, 0\rceil < 0$ and, since $\lceil 0, 0\rceil = 0$, it follows that p is not a best reply to $q^d(p)$. Step 2: every SPE prescribes p^* at r. By Lemma 3.2(ii), for all $p \leq p^*$, every best reply q at the node at which choosing p at r leads satisfies $q \geq q^s(p)$. Thus, by choosing $p \leq p^*$ at r, player s obtains payoff $\lceil p, q^s(p)\rceil$. As this value increases with p, the best reply at r is p^*. By Lemma 3.2(ii), every best reply q at the node at which choosing p^* at r leads satisfies $q \geq q^s(p^*) = q^*$.■

By Proposition 3.6, the game ${}_s\Gamma_M$ in which the producers' representative moves first has every backward induction path of play generating the market equilibrium outcome: the price is the equilibrium price of M and the quantity actually bought and sold is the equilibrium quantity of M. Interestingly, Proposition 3.6 does not hold for the game ${}_d\Gamma_M$ in which the consumers' representative moves first. In fact, let r be the root of ${}_d\Gamma_M$. No subgame perfect equilibrium prescribes $q > q^*$ at r: by Lemma 3.1(i), for all $q > q^*$, $p^s(q) > p^*$ is the best reply to q; but, by Lemma 3.2(i), $q^d(p^s(q)) < q$ is the best reply to $p^s(q)$. On the other hand, by Lemma 3.1(i), for all $0 < q \leq q^*$, $p^s(q)$ is the only best reply at the node at which choosing q at r leads. Hence, by choosing $q \leq q^*$ at r, player d obtains payoff $\langle p^s(q), q\rangle$, which need not be maximized at $q = q^*$ (since $\langle p^s(q^*), q^*\rangle > 0$ and, for all $p \in R$, $\langle p, 0\rangle = 0$, it follows that the best reply at r cannot be $q = 0$).

Example 3.7. Let M be a competitive market with $p^d = 24 - 4q$ as the inverse market demand function and $p^s = 2q$ as the inverse market supply function. The market equilibrium is $(p^*, q^*) = (8, 4)$ and $\langle p^*, q^*\rangle = 32$. But $\langle p^s(q), q\rangle$ is maximized at $q = 3$, with $\langle 6, 3\rangle = 36$. Thus, the only subgame perfect equilibrium of the corresponding game ${}_d\Gamma_M$ has d choosing $q = 3$ at r and s choosing $p = 6$ at the node that $q = 3$ links with r.

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